

Economies of Density and Congestion in the Sharing Economy*

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ABSTRACT

The sharing economy for a wide range of goods and services is expanding across the world. To direct the benefits from sharing capital services towards small-scale producers, governments in the developing world are increasingly intervening in fast-growing mechanization rental markets. However, the distributional and efficiency effects of these interventions are not well understood. Using a novel framework and newly collected data, we study an intervention in farm equipment rentals in India. We document large dispersion in rental rates, unused service capacity and delays in service provision. We then evaluate the impact of an increase in subsidized equipment supply in a calibrated model of frictional rental markets, optimal queueing and service dispatch. The constrained efficient allocation prioritizes large-scale producers because they economize on equipment travel cost; but small-scale producers are also valuable because they maximize machine-capacity utilization by increasing demand density. Through counterfactuals, we show that even when providers prioritize large-scale farmers, service finding rates for small-scale producers may exceed those of large-scale producers. The ability of additional supply to reach small-scale producers depends on the joint spatial and size distribution of demand, a novel rationale for the heterogeneous impact of subsidies to capital documented in the literature.

JEL codes: O13, O18, O47, R41.

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1 Introduction

The sharing economy for a wide range of goods and services is expanding across the world. This expansion has generated new interest in the potential to stimulate rental markets for equipment in developing countries, where small scale of production, low capital intensity and low capital quality are prevalent (Chen, 2020; Caunedo and Keller, 2020). These rental markets could be disruptive to agriculture in particular because they provide access to productive capital for small scale farmers, who conduct the lion’s share of farming in developing countries (Adamopoulos and Restuccia, 2014).

Startups are entering and creating mechanization rental markets across Asia and Africa to serve billions of small-holder farmers (Daum et al., 2021), often aided by government subsidies and tax incentives.¹ Proponents of government interventions in these markets point to the lack of mechanization in agriculture as evidence of frictions in access to capital, e.g. financial constraints; or of the unsuitability of engaging in large indivisible investments for farmers operating at small-scale, e.g. the purchase of a tractor. At the same time, there are concerns that subsidies to capital can be regressive, benefitting large-holder farmers, or granting market power to a handful of providers (Pingali et al., 1988). Despite the growing importance and increased government engagement in these markets, the efficiency and distributional impacts of increased equipment supply via government interventions are yet not well understood. Do increases in equipment supply disproportionately benefit large-scale farmers? Does efficiency in allocations favor rationing out small-scale farmers? Understanding whether there is a trade-off between access to capital and efficiency in equipment allocations is the focus of this paper.

To answer these questions, we combine a unique primary dataset with a novel model of frictional rental markets and optimal service dispatch. Our data comprise transaction-level information from the largest supplier of government-subsidized mechanization rental services in India, a census of 40,000 farming households characterizing demand and supply of rental services across 150 villages, and a survey of detailed farming activities from 7000 households

¹For instance, the Sub-Mission on Agricultural Mechanization (SMAM) in India disbursed \$304 million between 2014-2020 (or 0.01% of annual GDP) to several states for a range of mechanization initiatives, including establishment of custom hiring centers (CHCs) and distribution of agricultural equipment. China allocated a similar amount of GDP for farm equipment subsidies in 2020 alone.

which contains information on private rental markets as well as the joint spatial, productivity and farm size distribution for a subset of those farmers.²

We start by characterizing equipment rental markets in Karnataka, India, using our own Census of agricultural households. Karnataka is a particularly relevant state for this study because ownership rates of agricultural equipment were low prior to government intervention (less than 4% for tractors), and production scales are low (2.5 acres on average in our sample). We begin by documenting disparate engagement of small-scale and large-scale farmers in rental markets from private supply, with large scale farmers being more prevalent participants. Our survey of detailed farming activities shows that the most prevalent problem in accessing equipment rental services are delays in service provision, a more prevalent barrier than lack of credit or financing. Delays are disproportionately borne by small-scale farmers, but this disparity is partly driven by the spatial distribution of plots. When we control for village fixed effects, i.e. we compare plots in the neighbour of a village, the disparities across production scales are attenuated, with no evidence of differential delays between large- and small-scale farmers. We then bring in high-frequency data from ICRISAT to estimate the cost of these delays for farming output. We find that the average output per acre cost of delays within a 10-day window of the optimal planting date is 3.4% per day.

By 2016, the government of Karnataka had launched a private-public partnership with an objective to “assist the small and marginal farmers to provide machineries at their door steps”, i.e. increase mechanization access to small farmers. The intervention was designed to boost equipment supply in areas with low equipment ownership rates. Transaction-level data from the universe of Custom Hired Centers (CHC) set by the policy allows us to study queuing behaviour across farmers. This dataset includes information on machine-hours requested, acreage serviced, implement type and location. We document variation in queue lengths throughout the agricultural season consistently with synchronicity in equipment demand. We also document shifts in the composition of the queue between large-scale and small-scale farmers, which contributes to congestion in demand. While prices in CHCs are regulated not to vary throughout the agricultural season and across farmers, we document substantial price dispersion in informal rental arrangements.

²The survey of detailed farming activities is a representative sample of the average population in the Census in terms of average plot size, agricultural revenue, and crop choice.

The documented price dispersion and queueing behaviour motivates a model of frictional rental markets.³ We build a directed search model with two-sided heterogeneity, as in [Shi \(2002\)](#), and generalize it to accommodate multiple services within a period and service capacity constraints. The first feature allows us to discuss compositional changes in the serviced orders between large and small scale farmers as well as to optimize service provision in space. The second feature, paired with the discreteness in hours demanded, speaks directly to the role of small-scale orders in maximizing capacity utilization within a period.

The two sided heterogeneity in the market is driven by farmers’ scale of production (i.e. mechanization hours demanded); and by providers’ dispatch technology. We model dispatch technologies following two prevalent methods. There are profit-maximizing owners of equipment that travel around villages providing services, i.e. “market” providers.⁴ There are also CHCs that serve orders in a first-come-first-served fashion, so that order size and location are uncorrelated with the timing of service, i.e. labelled “fcfs” providers. This dispatch system may benefit small-holder farmers. Consistent with the data, providers can serve up to three orders within a day, posing an interesting combinatorics problem given the provider’s service capacity. This service capacity (hours per machine) is split between travel time and service time. In the model, providers set prices with commitment and farmers sort into available providers. Rental prices are allowed to vary by the scale of mechanization hours demanded but not by travel time. The reason is that empirically, we see no variation along this dimension;⁵ and technically, allowing contracts to vary with travel time when multiple orders can be served within the period requires making contracts contingent on the composition (along size and location) of those multiple orders. This feature would expand the set of rental contracts substantially.

The main predictions of the model are that when small- and large-scale farmers are equally distributed in space, small-scale farmers are more likely to approach the fcfs providers than the market provider. Conversely, large-scale farmers are more likely to approach market

³As highlighted by [Lagos \(2000\)](#) and [Sattinger \(2002\)](#), queueing models are powerful to micro-found a matching process between, in this case, farmers’ orders and service providers.

⁴It is also possible that owners may farm their own land and rent out their excess capacity, but given the low-ownership rates in our environment, we assume this source away.

⁵The reason for which we see little variation in prices with travel distance is likely that most of these markets are local, with equipment traveling no more than an hour from the CHC to meet demand in a given location.

providers. At the same time market providers have incentives to attract small-scale farmers to optimize capacity utilization, whereas fcfs have incentives to attract large scale farmers to obtain higher returns per time travelled. Sorting induces disparities in service finding rates between small and large-scale farmers but in equilibrium, rental rates adjust so that farmers face identical expected profits from either provider. The combination of price posting behaviour and directed search assures that allocations (queues) are constrained efficient [Wright et al. \(2021\)](#) and therefore, as we compare allocations, only distributional concerns are relevant.⁶ Hence, when assessing a potential equity-efficiency trade-off across allocations, we only consider differences in small-scale farmers' access to rental services.

We then assess the distributional impact of shifts in equipment supply, including the impact of the government imposed dispatch rules that intend to benefit small-holder farmers. There are three features that determine the distributional effects: (i) equipment moves in space to provide services, and this movement is costly; (ii) service capacity per machine is finite and idle capacity is costly for providers; (iii) agricultural production is highly time-sensitive and delays in provision are costly for farm output. These features generate economies of density because when farmers are closer in space they can be serviced faster than when farms are further away; and congestion in demand, because demand for equipment is synchronous. Servicing large-scale farmers lowers the cost of transport per hour serviced, so profit maximizing providers would like to prioritize those orders. At the same time, the discreteness in the size of plots and the provider costs from having idle machine capacity makes small-scale orders attractive, particularly if located in geographically dense areas.

First, we ask how does the current status quo equilibrium compares to allocations prior to the creation of CHCs, assuming that market providers accommodated the entry of subsidized providers (market providers are less than 5% of the total supply of equipment in these villages). We find that the subsidy generated a substantial increase in service capacity, which lead to two-to-three fold increases in service findings rates, and declines in the cost of

⁶As in [Shi \(2002\)](#), giving priority to large-scale orders among providers that have a technology to select farmers' types is optimal because those orders have the lowest marginal cost of service per unit of time traveled. To show that the equilibrium queues are socially optimal it suffices to show that providers set the expected value of market participation of the farmers to their social marginal return. This return takes into account the value of production for farmers, as well as possible crowd-out effects on other farmers queueing with a provider, i.e. longer delays for others.

service provision of more than 20% per hour serviced.

Second, we investigate the nature of the equilibrium when CHCs are allowed to dispatch orders like their market counterparts, i.e. a market deregulation that allows them to prioritize large-scale orders. The short run response of the economy (with no endogenous entry or exit of providers) implies higher service finding rates for large-scale farmers relative to the fcfs provision, consistent with the profit maximizing strategy of market providers. However, their service finding rates are below those of small scale farmers that are drawn to the market in response to the increased supply of services. While service findings rates for small-holder farmers do not change relative to the baseline, the rental costs increase for all farmers in the short run. In the long run, once the number of providers adjusts through entry and exit, rental costs fall again towards their baseline levels, and service findings rates are still higher for small holder farmers than large holder farmers.

Third, the service probability for small-holder farmers relative to that of large-holder farmers depends on the size of the subsidized supply of services. Only large enough increases in supply benefit small-holder farmers relatively to large-scale farmers. The level of the increase in supply that induces such an outcome depends on the joint spatial and size distribution of demand. Indeed, we show that market suppliers have comparable wait times to a fcfs service dispatch for plots that entail lower travel costs, irrespective of their size. The largest differences in service wait times, and productivity costs associated to delays across providers are concentrated in farms operating farther away from service hubs, i.e. where spatial density is lower.

These findings are drawn from a calibrated version of the model where we target the queue lengths for small-scale farmers, as observed in the transaction level data from the fcfs platform; the share of large scale farmers in each market, as inferred from the Census data; and the observed average profitability of fcfs providers. With these targets, we jointly calibrate the composition of farmers in each market, the ratio of farmers per provider and the cost of service provision as parameterized by the wage of equipment drivers. The main outcome of this calibration exercise are the endogenous queues by provider and farmer type, and the equilibrium rental prices per hour of equipment. We then add additional heterogeneity in farmers' demand in terms of equipment-hours as well as their locations. We bootstrap queues from the empirical joint distribution of equipment-hours demanded, productivity and plot

location to assess the merits of different dispatch systems in terms of expected wait-times and transportation costs. It also allows us to optimize routes in space given the realization of the queues.

Literature Review To our knowledge, this is the first paper to study the distributional consequences of rental markets for productive capital. In particular, we focus on heterogeneous access by scale of production and spatial allocation. A novel dimension of our problem is that service capacity constraints bring value to small-scale service orders because they improve capacity utilization. The value of small-scale producers in bringing density to demand has been mostly overlooked in the literature. We propose a tractable model that expands a common framework used in the search literature [Shi \(2002\)](#), along two relevant dimensions: multiple services and capacity constraints.

The notion that there might be scale economies associated to concentrating production in certain locations goes back to [Marshall \(1890\)](#). [Holmes and Lee \(2012\)](#) explore it in the context of crop choices of adjacent plots, where agglomeration economies rely on economies of scale in output. In our application, agglomeration economies stems from lower transport costs for service provision as in models of trade, [Rossi-Hansberg \(2005\)](#), as well as from the indivisibility of capital purchases, which generates incentives for sharing services through rental markets. [Duranton and Puga \(2004\)](#) review the micro-foundations for agglomeration economies and classifies them into three mechanisms: “sharing”, “matching” and “learning”. In our framework the first two mechanisms are at play. A paper that studies the “sharing” mechanism is [Bassi et al. \(2022\)](#), with an application to rental markets for door producers in urban Uganda, where they argue frictions are relatively limited. In contrast, we document substantial price dispersion in rental rates paired with unused service capacity, a common symptom of matching frictions. We are explicit about the role of “matching” in generating service transactions between providers and input demanders, and how they affect rental prices and queueing behaviour. Empirically, the ability to simultaneously observe supply of equipment services and queueing from the demand side of the market is an important advantage to discipline the properties of the matching process, as pointed out in [Brancaccio et al. \(2020\)](#).

Our problem is also distinct from previous studies in two important dimensions: agricultural production is time-sensitive, which makes demand synchronous, and delays are costly

for farm output.⁷ Delays are often overlooked as a barrier to technology adoption, yet they are a potentially important mechanism in sectors like agriculture, where returns are extremely sensitive to the timing of activities. The role of barriers to technology adoption in agriculture as a source of low productivity in poor countries has received extensive attention (see [Suri and Udry, 2022](#) for a review). We contribute to a growing literature that highlights the importance of adopting mechanized practices, [Caunedo and Keller \(2020\)](#); [Caunedo and Kala \(2021\)](#), by drawing attention to rental markets, which could be disruptive in lowering adoption costs relative to labor, ([Yang et al., 2013](#); [Yamauchi, 2016](#); [Manuelli and Seshadri, 2014](#)).

Finally, our quantitative results are of relevance to rationalize the seemingly contradicting and heterogeneous impact of mechanization policies throughout Africa, LatinAmerica and Asia. [Pingali \(2007\)](#) argues that “public sector run (...) tractor-hire operations have neither been successful nor equitable”. He suggests that mechanization attempts have failed because market infrastructure and economic incentives that induce production response were not there. Our theory and quantitative assessment suggests that the success of increases in equipment supply in improving accessibility to capital and ultimately, productivity, depend on the joint spatial and size distribution of farms. The role of geography for agricultural productivity has been studied by [Adamopoulos and Restuccia \(2021\)](#), but their work focuses on soil characteristics rather than on the accessibility to inputs of production.

2 Data and reduced form evidence

2.1 Data description

We combine four sources of data from the state of Karnataka for this project. Karnataka is one of the least mechanized states in India ([Satyasai and Balanarayana, 2018](#)). First, we use transaction level data from the universe of equipment rentals engaged through a public-private enterprise in Kharif 2018 (May-October). Second, we conducted a census of

⁷Other markets with similar features include the market for perishable products, or the allocation of personal shoppers. There is an extensive literature that studies taxi markets which also feature frictional meetings between customers and providers [Frechette et al. \(2019\)](#), however, costs bared by consumers from delays in service provision are typically abstracted away.

farming households covering 40,000 households across 150 villages, including information on equipment ownership and rental market engagement. Third, we collected detailed survey data covering 5500 farming households with information on ownership and rental market engagement, including information on equipment rental pricing, delays in service provisions, as well as output and input expenses, crop choices and land ownership. Fourth, we use high-frequency data from International Crops Research Institute for the Semi-Arid Tropics (ICRISAT), covering 6,200 plots in 18 villages in India during 2009-2014, with daily detailed measures of inputs and output in farming.

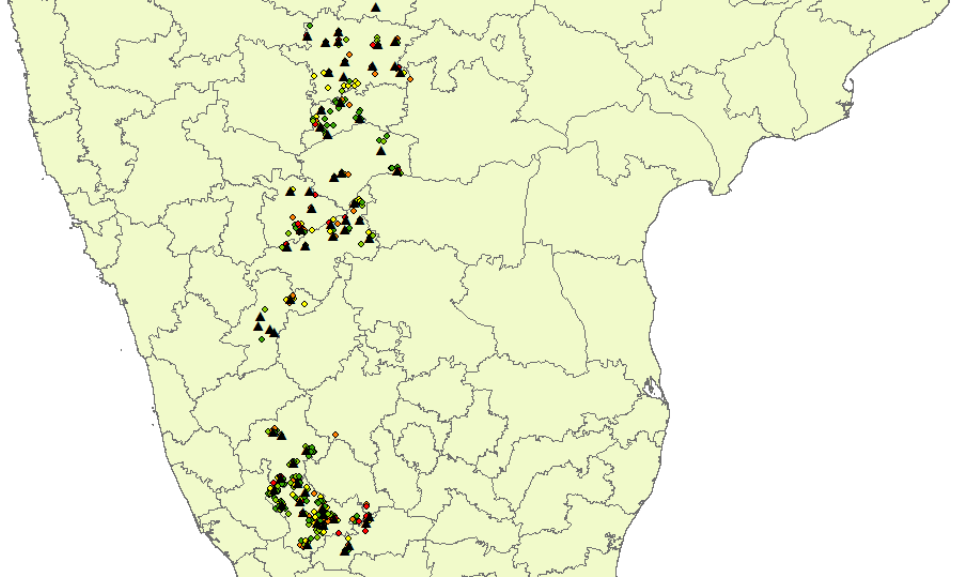
2.1.1 Mechanization platform data

In 2016, the state government partnered with the largest manufacturer of agricultural equipment in India to design and manage a platform through which farmers could rent equipment. As a public partnership, custom hiring centers (CHCs, also known as “hubs”) were set up in 25 districts throughout the state. Figure 1 plots currently active hubs in space. These CHCs provide rental services in nearby areas and farmers can access these services through a call-center, via an app on a smart-phone, or in person at the CHC.

Our administrative data consists of all the transactions completed through the platform since 2016 for about 60 hubs in Karnataka (27 hubs entered in 2016, 29 in 2018 and 10 in 2019). Over the time period covered by the data (October 2016-May 2019), over 17,000 farmers from 840 villages rented equipment from these CHCs. The data contains information on number of hours requested, acreage, implement type, as well as farmer identifiers (such as their name, village, and phone number). Equipment available varies across CHCs (hubs) but the median hub provides equipment that ranges from sprayers to rotavators. We focus our analysis on rentals of rotavators and cultivators, which are the pieces of equipment most commonly used at the land-preparation stage. Land-preparation is most likely to have been exposed to some form of mechanized practices in our sample (Caunedo and Kala, 2021).

Rental rates are fixed throughout the season in this platform, and regulated to be about 10% below market values. The service provision is first-come-first-served. When a service is fulfilled, a professional driver brings and operates the equipment at the farmer’s plot. Equipment arrives within a lapse of two days in most cases. If delays are longer, farmers

Figure 1: Locations of CHCs and Demand



Triangles indicate CHCs. We aggregate demand following the village where each farmer is registered. Green dots correspond to demand for the smallest plots (1 acre or less), red dots correspond to demand for the largest plots (4 acres or more).

are informed at the moment of booking (and may choose to cancel). Table 1 reports the 10 most commonly rented implements for the years 2017 and 2018, the number of transactions recorded for each implement, their per-hour rental price, and month where the implement is most commonly rented (has the highest number of transactions).

Table 1: Summary Statistics of Commonly Rented Implements from Rental Database

	Commonly Rented Implements		
	Number of Transactions	Median Price Per Hour	Peak Month
Rotavator 6 Feet	11,239	770	July
Cultivator Duckfoot	7,287	550	April
Cultivator 9 Tyne	5,245	525	May
Plough 2MB Hydraulic Reversible	3,716	450	February
Trolley 2 WD	2,436	250	January
Harvester Tangential Axial Flow (TAF)-Trac	2,048	1800	May
Rotavator 5 Feet	1,811	700	September
Blade Harrow Cross	1,793	360	March
Knapsack Sprayer 20 Litres	1,688	22.5	October
Blade Harrow 5 Blade	1,600	360	June

2.1.2 Survey and census data

We also exploit survey data collected in June-July 2019 around a representative sample of the equipment hubs in the main sample. The survey includes approximately 5,000 farmers, and asks for detailed information on input and output per plot, crop choices as well as a variety of details related to farmers' engagement in rental markets, and equipment ownership. We also ask about their perceptions on barriers to participation in the market.

To validate the the characteristics of the farmers in the survey relative to the population, we also run a census of agricultural farmers including more than 40,000 farmers in the villages covered by the survey data. This data contains a handful of questions related of equipment ownership which is helpful to construct measures of market supply.

2.1.3 ICRISAT data

We use data from a broader set of Indian states as recorded in ICRISAT's household-level panel data. This data contains high-frequency (daily) agricultural information, including, season-level agricultural operations, their timing, costs and total revenues. The data covers eighteen villages over 2009-2014 in Andhra Pradesh, Gujarat, Karnataka, Maharashtra, and Madhya Pradesh.

2.2 Motivating facts

We start by describing the characteristics of the service demand and farmers equipment supply. Then, we focus on a handful of outcomes that are informative to the theory that we describe in Section 3. First, because agricultural activities are highly time sensitive, the timing of demand is synchronous leading to endogenous waiting times as a function of service capacity. The service capacity includes farmers' ownership as well as CHCs capacity. Second, because equipment needs to travel for transactions to take place, the joint distribution between travel time and the scale of demand, i.e. equipment-hours per request, is a key input when optimizing service provision. Third, we document substantial price dispersion in rental rates after controlling for observable household characteristics and village/market characteristics, consistent with frictional rental markets. Fourth, delays in service provision

are costly to farmers, because they affect field productivity. In what follows we document each of these features.

2.2.1 Service Capacity and Service Demand.

We start by reporting patterns of ownership (service capacity by farmers) and rentals of equipment across the farmers in our survey (see Figure 2).⁸ Most farmers report owning hand tools and animal pulled equipment. Less than 10% of the farmers report owning larger equipment such as tractors, or rotavators and cultivators. At the same time, tractors and cultivators are among the pieces of equipment with the highest equipment-hours rented. The average hours rented in a season per farmer is 12 hours for tractors and 10 hours for cultivators. These rental transactions mostly entail relational contracts. We collect information on the typical customer for a farmer that rents out his/her equipment. We find that 72% of owners report to renting out to people they know from the village or with whom they have worked with in the past.

Delays are the most common issue faced by farmers when renting equipment, with 78% of farmers reporting it as an issue. Importantly, larger farmers (cultivating at the 75th percentile of the land size distribution) are nearly 5 percentage points less likely to report delays as an issue. Hence, delays in accessing mechanization are more pervasive among smaller farmers.

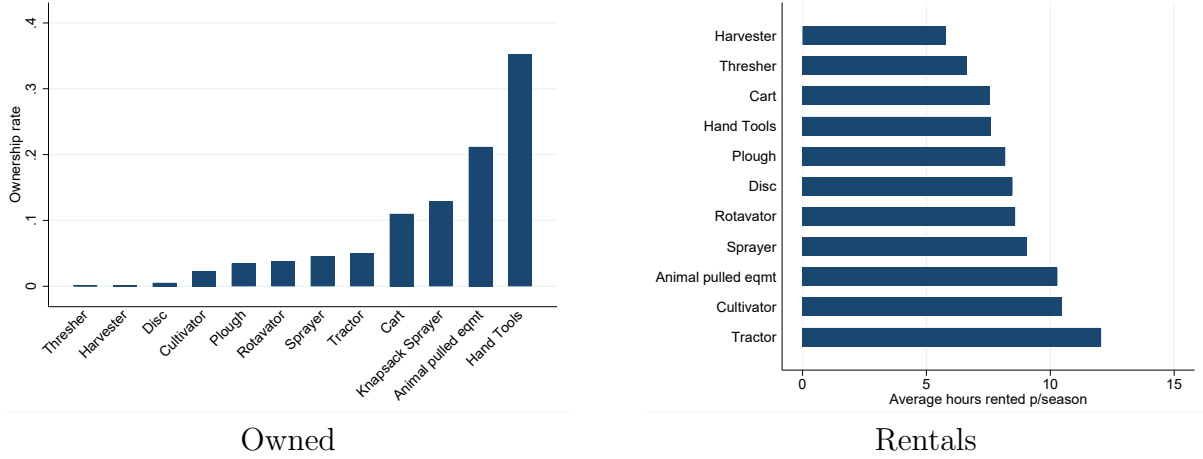
Given the disparities in value of agricultural implements as well as their contribution to production, it is useful to construct a measure of equipment services from rentals and owned equipment. We measure these services as the product of average hours of usage during a season h_i , market rental rates, r_i and the number of implements i owned or rented, N_i . Hence, equipment services in a farm k are

$$k = \sum_i N_i r_i h_i$$

The main hypothesis behind this measure is that differences in rental rates across implements shall reflect differences in the services they provide, and that therefore, more expensive equipment provides higher services to production. The main challenge in constructing such

⁸Appendix D reports similar statistics using data from the Census.

Figure 2: Ownership and rentals by implement.

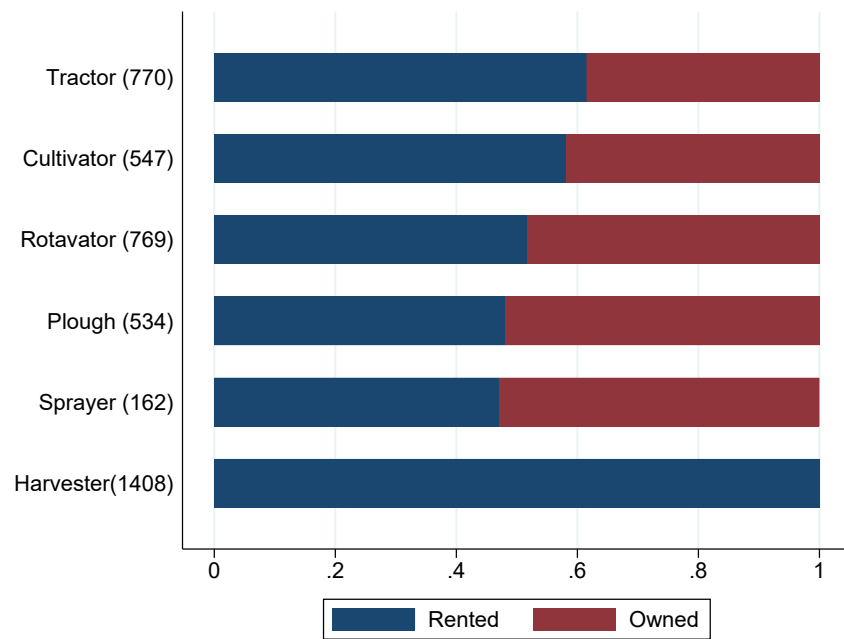


The ownership rate is the share of farmers that report owning a given implement relative to the total population surveyed. Rental hours correspond to the average hours reported for the whole season.

a measure is the availability of data on market rental rates. We exploit our transaction level dataset to construct mean rental rates per implement at the village level. Figure 3 displays log owned and rented services. Harvesters (the most expensive implement in our bundle) is reported to be only rented. For those farmers using tractors, more than 60% of the services available in the farming sector come from rentals whereas the remaining 40% stem from ownership. Services associated with smaller and cheaper equipment, such as sprayers, are equally accounted for by rentals and ownership.

It is worth noting that given land holdings, ownership of equipment is not cost-effective for most farmers. For instance, the rental price of a rotavator is between ₹750 and ₹1,000 per hour (including tractor, a driver and fuel) and the average farmer demands about 6 hours of rotavator services in the season or between ₹4500 and ₹6000 in services. The purchase price of a new rotavator is over ₹110,000 which means that, absent maintenance costs (which are certainly non-negligible), the average farmer needs 19 years to amortize the investment. The rental rate for an inferior technology that serves a similar purpose, i.e. a harrow, is half of the rental rate of the rotavator (₹360) and the cost of purchase is about ₹50000. Overall, these price differentials are consistent with the observed extensive engagement in rental markets for equipment.

Figure 3: Capital services from ownership and rentals, by implement.



Shares of log capital services by implement and ownership/rental status. Average rental rates for an hour of service (in ₹) are reported next to each implement.

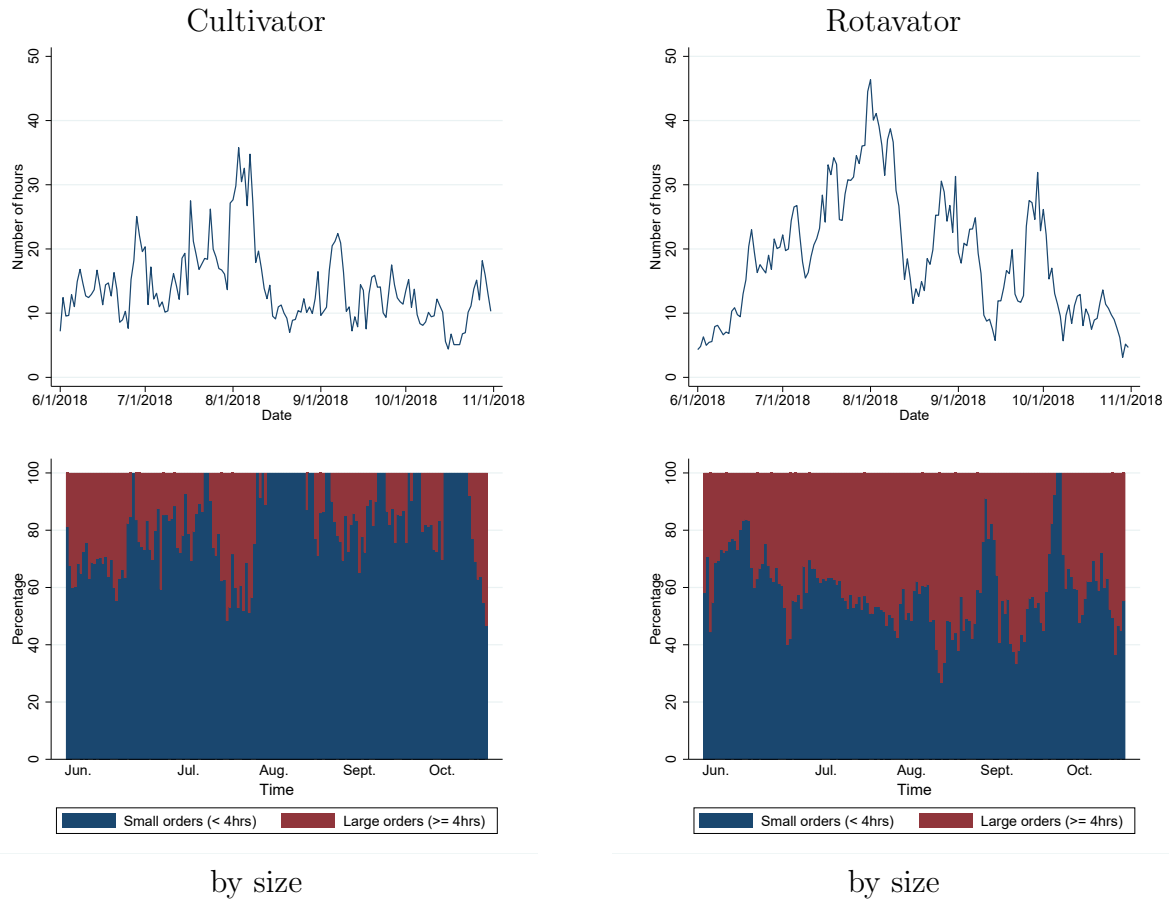
2.2.2 Heterogeneous Queuing by Production Scale

The demand for equipment rental services vary by agricultural process and therefore throughout the agricultural season. The synchronous nature of many of these processes across farmers induces queuing in the market. Our transaction level data allows us to measure demand fluctuations by computing hours outstanding for service at a daily frequency. We focus on two commonly rented implements for land preparation, rotavators and cultivators. Indeed, our survey data indicates that farmers are most likely to engage in the rental market for land preparation.

Figure 4 shows hours of unfulfilled orders for each of these implements over the 2018 kharif season. Queueing peaks by the end of July for rotavators and beginning of August for cultivators. At the peak of the season, the average provider faces 40 hours of demanded services in queue, which account for over 12 orders on average at a point in time.

Demand moves distinctively between large and small requests, measured in service hours (Figure 4). A large portion of hours outstanding are accounted for by small orders (less than

Figure 4: Hours outstanding in the queue.



Averages hours outstanding in the queue across hubs in Kharif 2018, overall (top panel) and by order size (bottom panel).

4 hours of service), although at peak time the share of hours accounted for by large farmers increases.

2.2.3 Delays in rental services

As demand fluctuates over the season in a somehow predictable manner, it is expected that service supply may adjust. If supply expands proportionally to the increase in demand, any delays in service supply could be constant across the season. We find that service rates fluctuate during the season, and that they positively correlate with hours serviced suggesting some adjustment in supply (see Appendix Figure 16). The relationship between hours in

the queue and service rates is non-linear, increasing for low service rates and declining for high-service rates, suggesting longer delays then. At peak queue hours, service rates in the CHC are 40% on average, suggesting that it takes 2.5 days to go through a hub’s queue.

Because our queue measures and service rates exploit data from the administrative platform only, we complement this analysis with delays reported by the farmers in our survey data. Service delays are negatively associated with cultivated area suggesting that even if the productivity costs of delays are of same magnitude between small- and large-holder farmers, the incidence of those delays is disproportionately borne by those with small plot sizes, columns (1) and (3) in Table 2. It is possible that these delays are explained by the geographical location of plots since equipment needs to travel to generate services. Columns (2) and (4) in Table 2 show that delays have an important spatial dimension, because adding village fixed effects substantially attenuates the coefficient on the log of land size, and increases the r-squared by eight or nine times (depending on whether only positive delays are considered, or all delays are included in the regression). That is, in the surroundings to a particular village, small and large farmers face similar delays, but if this clustering is not accounted for, smaller farmers face longer delays.

Table 2: Delays as a Function of Land Area and Location Fixed Effects

	Delays (Sum of Average Delays Over the Season)			
	(1)	(2)	(3)	(4)
Log(Area)	-0.215* (0.115)	-0.144 (0.0926)	-0.319** (0.145)	-0.128 (0.108)
Observations	5,615	5,615	4,345	4,345
R-squared	0.002	0.182	0.003	0.252
Village Fixed Effects	No	Yes	No	Yes
Mean Delays	2.158	2.158	2.789	2.789

Estimated coefficients from a regression of reported delays in service provision and the log(area) owned. The first two columns include those that report zero delays whereas the last two columns only focus on those that report positive delays.

2.2.4 How costly are these delays?

We define an optimal planting time as the date that maximizes the profits per acre in a given village year.⁹ ICRISAT’s high frequency data is particularly suitable for this exercise.

⁹The optimal measure is constructed using the approach in Kala (2017).

Then, we define the cost of the delay as the difference in average value added per acre or profit per acre (depending on the variable of interest) as we move away from the optimal planting date. Formally, we estimate

$$Y_{i,year} = \beta_0 + \beta_1^+ (\text{Planting Date-Optimal})_{>0} + \beta_1^- (\text{Planting Date-Optimal})_{\leq 0} + \alpha X_{i,year} + \epsilon_{i,year}$$

where X are controls for plot characteristics, farmer, village and time fixed effects. Standard errors are clustered at the village level. Our estimates for the costs in value added per acre are reported in Table 9. They indicate that within a 5-day windows, each additional day away from the profit maximizing date entails a cost of 3.4% in terms of value added per acre. In the 5-day window, for farmers that plant too early relative to the optimal date, moving *closer* to the optimal date by one day increases value added per acre by INR 391 per acre. Conversely, for farmers that plant too late relative to the optimal date, moving *away* from the optimal date by one day reduces value added per acre by INR 215 per acre. Therefore, moving closer to optimal date increases returns. This result is robust to enlarging the window around the optimal planting date, with an estimate cost of delay in the planting date of 8.5% per day.

2.2.5 Frictional rental markets

But why are there delays to begin with? Is this a consequence of low ownership rates and service capacity, or rather the consequence of frictions in the rental market that prevent farmers and providers to contract services when desired? There are two features of the market that indicate the presence of frictions in the rental market.

The first features is that the current supply of equipment seem adequate to serve market demand. To compute supply we turn to a Census of 150 villages from the same area, which includes information on over 40,000 farmers. We assume that the equipment has a catchment area of about 10km, since transporting equipment over large distances is time-consuming and expensive, particularly for farmers whose main activity is not equipment rentals.¹⁰ We also include machines available in the CHCs within each relevant catchment area. On average, the

¹⁰This 10km cutoff was decided based on conversations with our data partner but results are robust to enlarging the catchment area to 20km radius.

number of available cultivators within a 10 km radius can serve up to 2016 orders per season, while average demand is 1190 orders per season. For rotavators, available supply can serve up to 1008 orders in the season while market demand is 450 orders. In these computations we assume a six-week plant preparation season and that each piece of equipment serves three orders a day. The latter is consistent with serviced orders per equipment per driver at peak utilization in our transactions dataset.¹¹ Hence, these machine-hours supply and demand estimates within each geographical market suggests that congestion may not be related to supply shortages.

The second, and perhaps most important feature, is the presence of price dispersion in rental prices of equipment within a 10km catchment area of each village. As part of our survey we ask farmers how much did they pay for land-preparation equipment rentals during the season prior to Kharif 2018. Plant-preparation equipment includes mostly rotavators and cultivators, implements that rent out for similar hourly rates at the CHC. Figure 5 shows the distribution of rental rates paid per hour, controlling for village fixed effects. That is, the variation in rental rates per hour serviced across farmers within a village.¹² The interquartile range is 1.71 while the coefficient of variation is 0.67. [Burdett and Judd \(1983\)](#) were the first to show that price dispersion could arise in an environment with identical agents where consumers/farmers found it costly to search for providers. Price dispersion can also be related to informational asymmetries ([Varian, 1980](#)) or to consumer preferences for certain providers over others ([Rosenthal, 1980](#)). Overall, the exchange of identical goods for heterogeneous prices is typically a sign of frictions in the market, which we entertain through the structural model that we study next.

¹¹Even when assuming a shorter 4 week season, supply would account for 1344 orders per season for cultivators and 672 orders per season for rotavators, well under seasonal demand for each equipment.

¹²Observed dispersion is similar even after we account for total area cropped. Indeed, on average hourly rental rates are lower for farmers with larger cropped area. These results are available upon request.

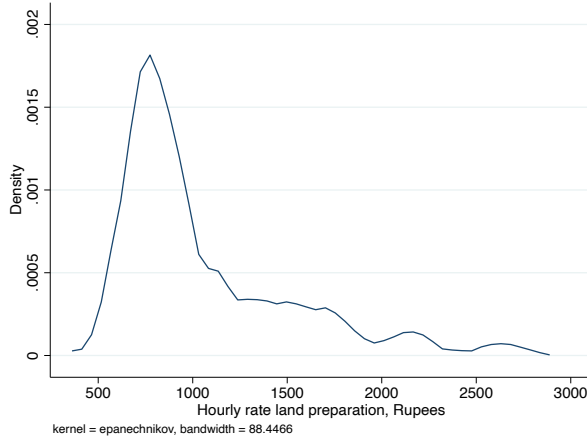


Figure 5: Rental rates

Notes: Hourly rates for equipment rented at land-preparation, which includes Cultivators and Rotavators. Source: Own survey data.

3 A frictional model of capital rental services in space

To build a quantitative assessment of the workings of these rental markets, we extend the two-sided heterogeneity framework of Shi (2002) along two dimensions. First, we allow for multiple orders to be served within each period. Second, we build in capacity constraints for service providers. The first feature is important to assess implications for travel times across orders, i.e. optimal service routes and the costs associated to them. The second feature is important to quantify congestion and the equilibrium service delays as a function of the composition of the service queue in terms of the size and geographical location of the plots being served.

3.1 Environment

Consider an economy populated by F farmers that use capital services for production. There are H service providers (machines) that provide capital services through frictional rental markets. The locations of farmers and service providers is given, and a market is defined as a catchment area around any of these providers.

Farmers differ in their demand for equipment-hours and in their geographic location. A fraction s of them are large-scale farmers and demand k_s hours, while the remaining $(1 - s)$ fraction are small-scale farmers, and demand k_{s-} hours. We think of the demand scale as

being determined by land-holdings. Farmer’s productivity gets realized once inputs have been committed, e.g., weather shocks or delays in equipment rental service provision that affect the output from the farm. These shocks induce revenue costs to farmers. For simplicity, we focus on ex-post shocks related to delays in service provision, which are idiosyncratic to the farm.

Service providers that can serve up to o orders a day and have the same service capacity, up to \bar{k} machine-hours a day, but their technology for service provision differs. A fraction h of providers use a first-come-first-served (fcfs) dispatch technology, while the remaining fraction $1-h$ has access to a selection technology that allows them to prioritize certain type of service requests (mkt).¹³ For simplicity, we assume no depreciation or capital accumulation and no maintenance costs.

Denote the ratio of farmers to service providers, $f = \frac{F}{H}$, and focus on the case where the market is large, i.e. $F, H \rightarrow \infty$ and neither side is infinitely larger than the other, $f \in (0, \infty)$. Providers post prices r_{ij} and a selection criteria (with commitment) simultaneously at the beginning of each period. Rental prices can depend on the scale of the plot serviced. Geographical considerations for service provision are included into the opportunity cost of moving equipment from a provider to the plot. Farmers decide whether and which provider to approach, generating queues for each available provider. Providers decide which orders to serve given their selection criteria and capacity constraint. Then, farming production takes place. Given the large number of providers and farmers we focus on a symmetric mixed-strategy equilibrium where ex ante identical providers and farmers use the same strategy and farmers randomize over the set of preferable providers. We assume that agents find it difficult to coordinate their decisions in a large market.¹⁴

A type i -farmer’s strategy is a vector of probabilities $P_i \equiv (p_{i,\text{fcfs}}, \dots; p_{i,\text{mkt}}, \dots)$ where p_{ij} is the probability of applying to each type j -provider. Those that request a service from the fcfs provider face the same probability of being first in line irrespective of the machine-hours demanded. A type- j provider’s strategy consists of rental rates per hour serviced, r_{ij} and

¹³Albeit h and H are assumed exogenous, both of them can be easily endogenized with a costly set up of providers and an associated free-entry condition.

¹⁴Lack of coordination in demand seems to be a prevalent feature of this market. Using the population relevant to these subsidies we piloted an incentive scheme inducing discounts in rental costs for farmer coordinating demand in time and space. We find little evidence of coordination.

a selection rule $\chi_j \in [0, 1]$ for the market provider. The selection rule applies only when a provider receives requests from both types of farmers, in which case the provider prefers the large scale farm if $\chi_j = 1$, prefers a small scale farm if $\chi_j = 0$, and he is indifferent between them for $\chi_j \in (0, 1)$. When the market provider receives requests from a single farmer type, he randomly selects one farmer for service. The fcs providers serve orders as they arrive in the queue.

Each farmer maximizes expected profits from farming trading off the probability of obtaining a rental service and the cost of such a service. The characteristics of their demand and geographical locations are as in Assumption 1.¹⁵

Assumption 1: *Equipment-hours demanded by small and large scale farmers satisfy $k_s > k_{s-}$. The expected travel time to servicing small-holder farmers is weakly higher than that for large-scale farmers, $d_s \leq d_{s-}$.*

The service capacity of providers satisfies Assumption 2.

Assumption 2: *Providers can serve at most o -orders within a day, and capacity satisfies*

$$o(k_{s-} + d_{s-}) \leq \bar{k} \quad \text{and} \quad o(k_s + d_s) > \bar{k},$$

$$(o - 1)(k_s + d_s) + k_{s-} + d_{s-} \leq \bar{k},$$

$$(o - 1)(k_{s-} + d_{s-}) + k_s + d_s \leq \bar{k}.$$

Hence, if the provider serves only large-scale orders, it can serve $(o - 1)$ orders, or it can instead combine those $(o - 1)$ orders with one small scale order. Finally, we assume enough capacity to serve $o - 1$ small scale orders and one large scale order.

We assume the empirically relevant service capacity for each provider: $o = 3$ orders can be fulfilled within the period. This value is consistent with the median number of orders served within a day in our administrative data.

¹⁵The capital demand for a farmer is given exogenously. We could trivially model the link between land-holdings and capital demand through a Leontief production function between capital and land. The key assumption is that the decision to mechanize is either taken for the entire plot, or not at all.

3.2 Queue lengths as strategies

A convenient object for analysis is the *queue length*, i.e., the expected number of farmers requesting a service from a given provider.¹⁶ Let q_{ij} be the queue length of type i farmers that apply to a type j provider, where $i \in \{s, s^-\}$ and $j \in \{\text{fcfs}, \text{mkt}\}$. Then, $q_{sj} = sFp_{sj}$ and $q_{s^-j} = (1-s)Fp_{s^-j}$. The probability of approaching different providers for a single farmer should add up to one, which leads to the following feasibility constraints

$$H(hq_{s,\text{fcfs}} + (1-h)q_{s,\text{mkt}}) = Fs \quad (1)$$

$$H(hq_{s^-, \text{fcfs}} + (1-h)q_{s^-, \text{mkt}}) = F(1-s) \quad (2)$$

A farmer of scale i that requests service from provider j gets served with probability Δ_{ij} . This conditional probability depends on the provider's selection criteria, its capacity, machine-hours demanded k_i and the expected travel time for service d_i . Hence, Δ_{ij} is the sum across all possible number of orders of type i being served, \bar{o}_i , of the probability of servicing \bar{o}_i type i farmers, $\phi_{ij}(\bar{o}_i)$, times the probability that a certain farmer of type i is chosen, $\tilde{\Delta}_{ij}(\bar{o}_i)$,

$$\Delta_{ij} = \sum_{\bar{o}_i \in \{1,2,3\}} \phi_{ij}(\bar{o}_i) \tilde{\Delta}_{ij}(\bar{o}_i).^{17} \quad (3)$$

The probability of type i being served (weakly) declines in the queue length of type $i' \neq i$ farmers. For the first-come-first-served provider the result is straightforward because service probabilities decline with the number of machine-hours in the queue, irrespective of their type. For the market provider with a selection criteria that favours type i' farmers, the decline in the probability of service for type $i \neq i'$ is strict as the number of type i' farmers in the queue increases. The service probability for type i' farmers is independent of the queue length of type $i \neq i'$ due to the selection criteria.

¹⁶From a theory standpoint, when the number of providers and firms grow large, the probability of requesting a service to a given provider approaches zero and it is inconvenient to work with. From an empirical standpoint, queues are observable in our administrative data, while probabilities are not.

¹⁷The full derivation can be found in Appendix A.

3.3 Market cost of provision.

We follow [Burdett et al. \(2001\)](#) and describe a farmer's decision as a function of the market price it would get for the rental service, r_{ij} , which in turn determines its expected "market" profit, U_i . Farmers take the value of the market profit as given when the number of agents in the economy is large, $F, H \rightarrow \infty$. Each farmer chooses a service provider to minimize costs given U_i and the production technology:

$$\min_j r_{ij} k_i$$

subject to

$$\pi_{ij}(z_{ij}, k_i, r_{ij}) \equiv \Delta_{ij} (E(z(\Delta_{ij})) k_i^\alpha - r_{ij} k_i) \geq U_i,$$

where π_{ij} are the expected profits of the farm when requesting service from provider j and $z_{ij} \equiv E(z(\Delta_{ij}))$ is the expected productivity in the farm, possibly a function of expected service probability. Farm's productivity depends on the realization of a random shock that yields the timing of agricultural activities. We summarize the optimal timing for agricultural activities by the optimal "land preparation" date, θ^* , and relate deviations from this optimal timing to productivity costs. The realization of the land preparation date is a random draw, θ , from a known distribution $G(\bar{\theta}(\Delta_{ij}))$ with mean $\bar{\theta}(\Delta_{ij})$ that depends on provider j 's probability of service. We assume that $\frac{\partial \bar{\theta}(\Delta_{ij})}{\partial \Delta} < 0$ so that a high probability of service induces shorter average wait time to service. If the realization of the preparation date differs from the optimal, the farmer faces a productivity cost proportional to the delay relative to the optimal date as follows, $z = \bar{z}(1 - \eta(\theta - \theta^*)I_{\theta^* \leq \theta})$, where η is the productivity cost per delayed service day in percentage. Expected productivity is

$$E(z(\Delta_{ij})) = \bar{z}(1 - \eta(\bar{\theta}(\Delta_{ij}) - \theta^*)I_{\theta^* \leq \bar{\theta}(\Delta_{ij})}).$$

The expected productivity is independent of the choice of provider whenever the expected wait time is relatively low, i.e. the probability of service is high. Finally, because the draw of the service provision is idiosyncratic, there is no aggregate uncertainty in the economy and factor prices are time independent.

A type i farmer requests a service from a type j firm with positive probability if the

expected profits are weakly larger than U_i . The strict inequality cannot hold because then a type i farmer would apply to that provider with probability 1, yielding $q_{ij} \rightarrow \infty$ as the number of farmers grows large. Then, $\Delta_{ij} \rightarrow 0$ contradicting that $\pi_{ij}(z_{ij}, r_{ij}, k_i) > \tilde{U}_i$.

The farmers' strategy is

$$q_{ij} \in (0, \infty) \quad \text{if} \quad \pi_i(z_{ij}, r_{ij}, k_i) = \tilde{U}_i \quad (4)$$

$$q_{ij} = 0 \quad \text{if} \quad \pi_i(z_{ij}, r_{ij}, k_i) < \tilde{U}_i \quad (5)$$

This expression summarizes the tradeoff between lower provision cost and higher farming profits; and a lower probability of service. Given the shape of the probability function (which enters into expected profits, π) there exist a unique queue $q(r_{ij}, U_i)$ that satisfies the problem of the farmer. The farmer decides his queueing strategy as a function of his capital demand, k_i , expected productivity z_{ij} and market prices r_{ij} .

3.4 Service Providers

A service provider with capacity j maximizes expected returns. The cost of servicing a farmer depends on its location relative to the provider. The location of each plot is indexed by d_i and $\mathbf{d}_{\hat{q}}$ is a vector collecting the locations of the orders completed within a period, $\hat{q} \in q$. Providers choose the cost of service r_{ij} taking the the machine-hours demanded by each type of farmer as given. For a vector $\{U_i, k_i, d_i\}_{i=s, s^-}$, he chooses the queue lengths by picking the cost of service and service strategy. The cost of travel time includes the foregone services that could have been provided if the equipment was not travelling as well as the opportunity cost of the driver, which commands a wage w per hour. Finally, the queue length is reset each period and therefore the service provision problem is static.¹⁸

Consider the problem of a first-come-first-served provider. His value is the expected return from servicing at most $o = 3$ orders within each period. Let \bar{o}_i be the number of orders of type i being served within the period. The per period return \tilde{V} from facing queue q_{fcs} depends on the number of orders of each type being served, $\{\bar{o}_s, \bar{o}_{s^-}\}$ and the revenue

¹⁸This feature allow us to handle the high dimensionality of the combinatorics problem when providers are allowed to prioritize certain farmer types.

per type net of labor and transportation costs, $\{r_{i,\text{fcfs}}k_i - wk_i - wd_i\}$.¹⁹ The value for a first-come-first-served provider is

$$V(\bar{k}) = \max_{\{r_{i,\text{fcfs}}\}_{i=s,s^-}} \tilde{V}(\{\bar{o}_s, \bar{o}_{s^-}\}_{q_{\text{fcfs}}}, \{r_{i,\text{fcfs}}k_i - wk_i - wd_i\}_{i=s,s^-}), \quad (6)$$

subject to farmers' strategies, equation 4, and feasibility

$$\sum_{i \in q_{\text{fcfs}}} k_i + d_i \leq \bar{k}. \quad (7)$$

Consider now the problem of a market provider who, in addition to choosing the cost of provision, r_{mkt} , chooses a selection criteria χ . This choice in turn determines the type of orders being served and their quantity, given service capacity. The value of a market provider is

$$V(\bar{k}) = \max_{\chi, \{r_{i,\text{mkt}}\}_{i=s,s^-}} \tilde{V}(\{\bar{o}_s, \bar{o}_{s^-}\}_{(q_{\text{mkt}}, \chi)}, \{r_{i,\text{mkt}}k_i - wk_i - wd_i\}_{i=s,s^-}), \quad (8)$$

subject to farmers' strategies, equation 4, and feasibility

$$\sum_{i \in q_{\text{mkt}}} k_i + d_i \leq \bar{k}. \quad (9)$$

The full description of the value of these providers, \tilde{V} , can be found in Appendix B.

4 Symmetric Equilibrium

A symmetric equilibrium consists of farmers expected profits U_s, U_{s^-} , provider strategies r_{ij}, χ , and farmer strategies, q_{ij} for $i = \{s, s^-\}$ and $j = \{\text{fcfs}, \text{mkt}\}$, that satisfy:

1. given U_s, U_{s^-} and other providers' strategies, each type provider maximizes value, equations 6 or 8;
2. observing the providers' decisions, farmers choose who to queue with, equation 4; and

¹⁹We assume the revenue is separable in the number of orders and relax this assumption in the quantitative exercise when the provider minimizes transportation cost across orders.

3. the values U_s, U_{s^-} , through q_{ij} , are consistent with feasibility, equations 1 and 2.

Proposition 1. *In all symmetric equilibria where providers serve both types of farmers, the selection process is $\chi = 1$ and the per period profit of servicing farmers of type i is V_i^j :*

$$V_i^j = \gamma_{1i}^j(\tilde{z}k_s^\alpha - wk_s - wd_s) + \gamma_{2i}^j(\tilde{z}k_{s^-}^\alpha - wk_{s^-} - wd_{s^-}),$$

where $\gamma_{1,i}^j, \gamma_{2,i}^j$ are non-linear functions of the queue lengths and the elasticity of the service probabilities with respect to the length of the queue and $\tilde{z} \equiv \bar{z}(1 - \eta)$.²⁰

The expected per period value of servicing large-scale farmers is higher than for low-scale farmers, $V_s^j > V_{s^-}^j$. If the surplus from large-scale orders is sufficiently larger than from small-scale orders, the expected profit for large-scale farmers is greater than for small-holder farmers, $U_s > U_{s^-}$.²¹

A few characteristics are worth highlighting. First, differences in location and the cost of travel explain disparities in the incentives to serve farmers operating different scales. For two plots located at the same distance to the provider, the marginal cost of service is lower for larger scale farmers. Second, small-scale farmers are useful in terms of capacity utilization (Assumption 1) and therefore, even providers that prioritize large-scale farmers have incentives to serve them. Third, the fcs provider manages to attract some large-scale farmers by lowering their rental costs relative to the mkt provider. These lower costs for both large and small farmers compensate them for higher expected queues at the fcs provider. Finally, the farmers' expected profit from equipment services depends on the return to his own demand for services and on the equilibrium rental rates. In equilibrium, farmers that are served by both providers shall be indifferent between them. Hence, the product between the probability of service, conditional on machine-hours demanded, and the cost of service should equalize across providers.

²⁰The expected productivity \tilde{z} is a log-linear function of an exogenous component \bar{z} and an endogenous component $1 + \frac{\partial z_{ij}}{\partial \Delta_{ij}} \frac{\Delta_{ij}}{z_{ij}}$ which depends on the elasticity of productivity to the probability of service. Under Assumption 3 in Appendix B this elasticity is constant and equal to $-\eta$. This assumption guaranties that the surplus from transactions is independent of the probability of service.

²¹The ratio of the surpluses $\frac{\tilde{z}k_s^\alpha - wk_s - wd_s}{\tilde{z}k_{s^-}^\alpha - wk_{s^-} - wd_{s^-}}$ must be larger than a constant that depends on the elasticity of the probability of service, see equation 24 in Appendix B.

5 Government regulation in equipment rental markets

In this section, we bring the model to the data to characterize how allocations change with the presence of a fcfs dispatch system relative to the market dispatch system, both in space and across farmers of different production scales. The key outcomes of interest are the selection of farmer types across providers, the equilibrium delays and therefore farming productivity costs, as well as provider profitability. We then ask whether small scale farmers are hurt by this deregulation despite efficiency improvements in allocations. Our market deregulation consists of (i) allowing first-come-first-serve providers to have access to a service selection technology, and (ii) allowing providers' entry and exit in the market mimicking the "long-run" equilibrium for the deregulated market. We show that the increase in capital supply from the subsidy is large enough to generate a disproportionately higher increase in service probability for small-holder farmers relative to large-scale ones. The relatively higher service probability for small-holder farmers is however not warranted for every level of the subsidy: only large enough subsidies benefit small-holder farmers relatively more.

5.1 Bringing the model to the data

The quantitative assessment of the impact of the government intervention in the rental markets for equipment consists of two blocks. The first block solves the model in Section 3 for the equilibrium market rental rates and queue lengths, given the empirical supply and demand for equipment services. The second block simulates queues and service provision strategies for farmers with different scales and geographical locations.

Solving the first block requires taking a stand on the heterogeneity in machine-hours demanded. We construct two groups of farmers following their average machine-hours requests in the transaction data: those with requests of more than 3.5 machine-hours per order are denominated large-scale while those with requests of less than 3.5 machine-hours are denominated small-scale. Then, we solve for an equilibrium in which both types of farmers are served by both types of providers, as in the data. We call this equilibrium the "status quo".

The second block involves finding the expected delay and subsequent productivity costs as well as provider profitability under alternative dispatch systems using equilibrium rental

rates and queue lengths from the first block. In theory, the queue length itself yields the expected wait time by farm type. However, we recognize that empirically, farmer heterogeneity is richer than the one accommodated by the stylized theoretical model both in terms of machine-hours demanded and in the spatial allocation of demand. We simulate 1000 paths of queues of length q^* and composition (q_s^*, q_{s-}^*) as dictated by the equilibrium of the selection model. The sample paths for queues (q_s^*, q_{s-}^*) are drawn from the joint empirical distribution of machine-hours and geographical location. Then, given the equilibrium rental rates and the technology for dispatch, we let the provider optimize service delivery. The optimization of service provision in space is effectively the solution to a traveling salesman problem, conditional on the set of orders in the queue.

5.1.1 Parameterization

There are 10 parameters per hub that need to be calibrated, as shown in Table 3. Eight of these parameters are calibrated directly from the data while the remaining two are calibrated internally by solving the model. Consistently with the evidence in Section 2 we use data for the Kharif season (June to October) in year 2018. We exploit four sources of data: (1) detailed transaction data from the government subsidized service provider, (2) our own survey of farmers, (3) our own census of farming households in the catchment area of the subsidized service providers and (4) high-frequency data from ICRISAT.

From those parameters measured in the data, four of them are common across hubs: the providers' discount factor β , and their opportunity cost of moving equipment in space w , the productivity cost of delays η , and the curvature of the farming profit function α . The remaining 6 parameters are hub specific and include the share of first-come-first-serve providers relative to the total supply of equipment in the catchment area of a hub h , the parameters characterizing the joint distribution of productivity and machine-hours demand within the catchment area of the hub (i.e., mean and standard deviation of productivity and the correlation between productivity and machine-hours), the ratio of farmers demanding service to the providers in the catchment area of each hub f , as well as the share of large farmers in the population of farmers demanding equipment in the catchment area of the hub s . The latter two model-calibrated parameters are chosen to match the queue length of small-holder farmers at first-come-first-served providers, and to make sure the equilibrium

Table 3: Parameterization

Parameter	Description	Value	Source/Moment
Measured directly in the data			
<i>common across hubs</i>			
α	Curvature of the profits function	0.6	Survey data
β	Discount factor	0.999	Interest rate
w	Travel/op. cost (INR/hr)	75	Platform data
η	Productivity loss/day	3.4%	ICRISAT sample
<i>hub specific</i>		method/value	
h	Share of fcfs providers	Table 4	Census data
μ	Log-normal mean of productivity	MLE	Survey data
σ	Log-normal s.d. of productivity	MLE	Survey data
ρ	Correlation order size and productivity	Table 4	Survey + Platform data
k_i, d_i	Joint-distribution of order size and travel time	B-splines	Platform data
\bar{k}	hub-capacity (hours)	Table 4	Platform data, peak
Calibrated using the model (hub-specific)			
s	Share of large farmers	Table 4	Census data
f	No. of farmers/No. of equipment	Table 4	Small-scale queue, fcfs

Notes: Benchmark model parameterization. Productivity is measured as output per acre. Hub-capacity corresponds to the hours serviced per machine within a day at the peak of service demand, i.e. the maximum number of hours outstanding in the queue during the season.

displays positive queues of small and large-scale farmers with both providers, as we observe in the data. In addition to these 10 parameters, we feed the distribution of plots in space (and their corresponding travel-time) as measured from the platform data.

We set the discount factor to $\beta = 0.999$ with an implied daily discount rate of 0.1%. The opportunity cost of travel time equals the hourly wage of a driver which is directly observed from the platform data, at $w = ₹75$. The curvature of the profit function is set to 0.6, as estimated from our own survey data on farm profitability. We exploit the fact that farming profits are proportional to this parameter, i.e. $\pi_i = (1 - \alpha)y_i$ and estimate α from the average ratio of profits to value added as reported by farming households.

To discipline the productivity costs of delays, $\eta = 3.4\%$, we use high frequency data from ICRISAT as described in Section 2. We also need to calibrate the optimal planting date, θ^* . We assume there is enough service capacity in the market that allows farmers to choose providers in advance in a manner that on expectation, there are no productivity losses from

using mechanized services, i.e. $E(z(\Delta_{ij})) = \bar{z}$.²² Once queues are realized, productivity costs realize as a feature of the service provision process. Finally, we need a mapping between the probability of service and the realization of the service date, $\theta(\Delta_{ij})$. We set it to be a strictly monotonic function, i.e. $\theta(\Delta_{ij}) = -\ln(\Delta_{ij})$. Its logarithmic shape brings tractability to the problem because it implies a constant elasticity of the delay time to the probability of service equal to the productivity costs of delays η , see Appendix B.

Then, we calibrate hub-specific parameters. We use our census, to compute the share of machinery available from government-subsidized hubs and that available from machine-owners (i.e. we count inventory of implements per hub and implements owned by farmers within the catchment area of each hub). To characterize the productivity of farmers requesting different machine-hours we use the subsample of transactions that overlaps with the survey data (approximately, 1,300 observations) and compute the underlying correlation between farm productivity, measured as output per acre, and machine-hours requested. Their correlation ranges from -0.28 to 0.35 displaying the wide-heterogeneity in demand characteristics across hubs, column (5) in Table 4. When machine-hours requested are proportional to plot sizes a negative correlation between output per acre and machine-hours follows from the negative correlation between productivity and farm size, as has been documented by others in the literature, e.g. Foster and Rosenzweig (2022). A positive correlation is consistent with more productive mechanized farms. Our data is rich enough to display both patterns. We assume that the distribution of productivity is log-normal, $\ln(\bar{z}) \sim \mathcal{N}(\mu, \sigma)$ and fit the empirical distribution of value-added per acre for survey farmers in the catchment area of each hub via maximum likelihood. The estimated mean of productivity suggests differences in log productivity across hubs of 36% (from 7.4 to 9.3) on average, and a log-variance ranging from 1.1 to 2.9, columns (3-4) in Table 4. Finally, we fit the joint distribution of machine-hours demanded and travel time to services from the platform data for each hub using B-splines, akin to a non-parametric estimation of the distribution, see Figure 17 in Appendix D. On the travel dimension, the distribution is typically bimodal, with orders bunching at less than 10-minutes travel time from the hub and 30-minutes travel time.²³

²²This is analogous to assuming an outside option that entails no usage of mechanization services and that is large enough so that farmers only participate in this market if on expectation, they face no productivity costs.

²³We could have alternatively calibrated a joint distribution of productivity, machine-hours requested and travel time. However, the overlap of the survey data and platform accounts for 20% of the survey data and

Table 4: Hub specific characteristics.

Hub (1)	Measured Directly					Calibrated	
	Supply		Demand			<i>Farmers</i>	
	sh. fcfs h (2)	capacity \bar{k} (3)	Productivity mean (4)	variance (5)	Correlation prod - hours (6)	<i>sh. large</i> s (7)	<i>per provider</i> f (8)
1	0.80	4	9.83	1.12	-0.01	0.3	5.0
2	0.86	4	8.89	1.33	-0.12	0.3	4.6
3	0.86	5	8.67	1.49	0.07	0.3	4.4
4	0.86	5	9.08	1.52	0.07	0.3	5.2
5	0.86	6	9.35	1.08	-0.20	0.5	3.3
6	0.86	6	9.08	1.52	0.07	0.5	3.3
7	0.86	7	9.08	1.52	0.07	0.4	5.3
8	0.63	8	8.85	2.56	0.16	0.4	4.1
9	0.75	11	8.85	2.56	0.16	0.4	3.4
10	0.67	11	8.15	2.89	0.01	0.4	4.7
11	0.75	14	8.85	2.56	0.16	0.3	3.4
average	0.79	7	8.97	1.83	0.04	0.4	4.3

Notes: Hub-specific parameters for each hub-implement combination, “Hub” in Column (1). Hubs labeled 3, 4 and 8 correspond to Cultivators while the remaining hubs contain information for Rotavators. Information for hubs labeled 4-5 correspond to different implements in a single government subsidized hub, and therefore demand characteristics are the same. Column (2) reports the share of first-come-first-serve providers in the total equipment supply within each catchment area. Columns (3)-(5) report demand characteristics for each hub, including the characteristics of the distribution of productivity across farmers and its correlation between hours demanded. Columns (6)-(7) report parameters calibrated jointly in the model.

Table 5: Moments

(1)	Share of large scale s		Queue q_{s-fcfs}		Queue <i>untargeted</i> q_{sfcs}/q_{s-fcfs}	
	data	model	data	model	data	model
	(2)	(3)	(4)	(5)	(6)	(7)
1	0.29	0.45	1.5	3.0	0.5	1.4
2	0.25	0.30	2.3	3.5	1.2	3.2
3	0.19	0.30	3.3	3.3	3.3	2.9
4	0.28	0.30	4.0	4.0	4.0	3.4
5	0.12	0.40	1.3	2.0	0.7	1.8
6	0.35	0.40	2.0	2.0	2.0	1.8
7	0.19	0.25	7.3	4.3	1.5	3.8
8	0.35	0.50	1.0	2.5	0.3	2.2
9	0.39	0.40	0.7	2.0	0.2	1.9
10	0.31	0.30	2.0	4.0	0.5	4.1
11	0.28	0.35	1.0	2.0	1.0	2.0

Notes: Calibration moments, data and model counterparts, Columns (2-5). Untargeted queue length for large-scale farmers relative to small-holder farmers, Columns (6-7).

Parameters calibrated jointly include the ratio of farmers to providers in the catchment area of each hub, as well as the the share of large-scale requests in that catchment area. The ratio of providers to farmers minimizes the distance between the model predicted queue of small-holder farmers at the fcfs provider and the data. We pick the share of large-scale requests that its closest to its empirical counterpart while generating an equilibrium allocation that displays service request from both types of farmers to both type of providers (as in the data). To do so, we take the stand that small-holder farmers are those with requests of up to 3.5 machine-hours per order, and large-scale farmers are those with orders of more than 3.5 machine-hours per order.

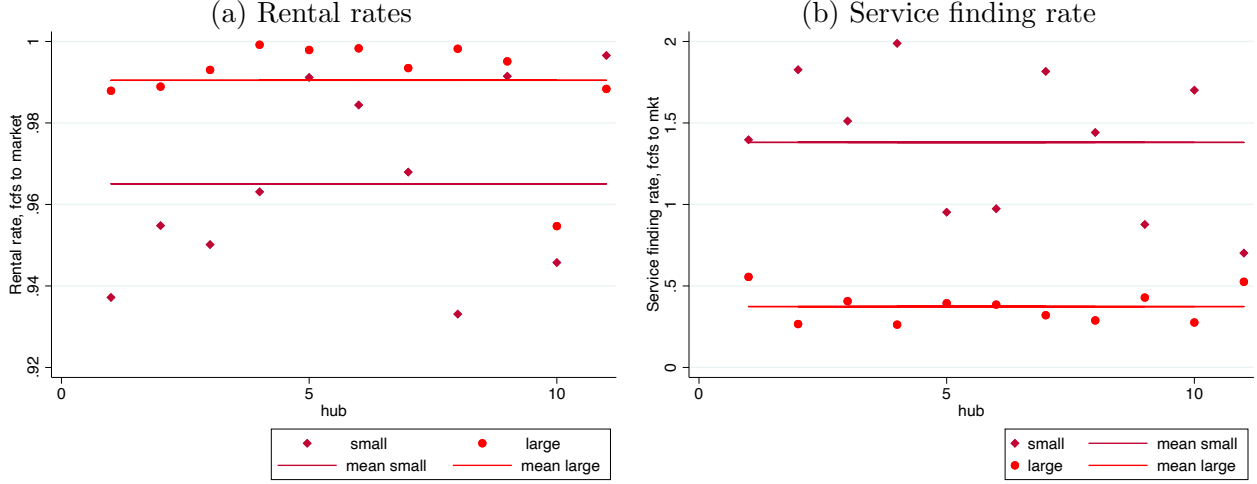
The calibrated ratio of farmers to providers ranges from 3.3 to 5.3, where a provider should be interpreted as a piece of equipment, column (8) in Table 4. The calibrated shares of large-scale farmers are higher than in the data (0.4 in the model vs. 0.27 in the data on average across hubs).²⁴ When there are few large-scale requests, the model generates queue lengths for small farmers that are broadly in line in the data, Table 5. Queues that are “too short” (less than 2 orders) fail to generate equilibria where both type of farmers request service from both providers because a queue length of 1 order for the market provider implies that small farmers are served with probability one there, given capacity. If the queue length is instead “too long” (more than 5 orders) the model benefits an equilibrium where small-farmers only request service from fcfs providers, which is inconsistent with the engagement of farmers across both types of providers, which we observe in the data.

For completeness, we report the (untargeted) ratio of queue lengths of large-scale and small-holder farmers. On average, this ratio is lower in the data than in the model, i.e. small scale farmers are more strongly sorted into fcfs providers than predicted by the model. This difference is in part driven by a larger share of large-scale farmers in the model than observed in the data, to be able to sustain equilibria where both types of farmers reach out to both providers.

we therefore benefitted including the fullness of the distribution of machine-hours and travel time. The latter is a key input into the costs of service and therefore the incentives to service large and small scale farmers.

²⁴Alternatively, we could have targeted the queue of large-scale farmers in the first-come-first-serve providers which we currently report as an untargeted moment. Results are qualitatively similar to those reported here and available upon request.

Figure 6: Equilibrium



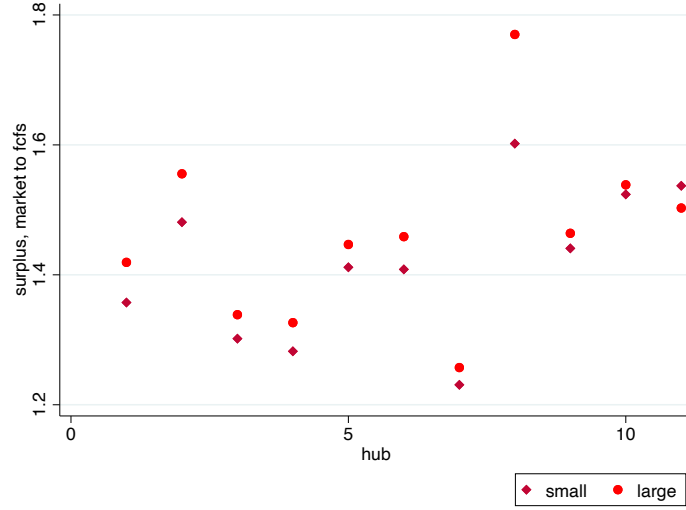
Notes: Equilibrium rental rates and service findings rates by hub for the calibrated economy. Hubs are ordered by increasing service capacity as in Table 4.

5.2 Status quo equilibrium

We solve for the rental rates and queue lengths when both types of farmers have access to both types of providers, i.e. the status quo equilibrium. In the remainder of the analysis we order hubs by service capacity \bar{k} in increasing order. The rental rates for both types of farmers are lower at the fcfs provider than at the mkt provider to compensate for the longer queues, Figure 6 panel (a). Rental rates are particularly lower for small farmers queueing with fcfs providers, which makes these providers attractive. Consistently, the service finding rate for small-scale farmers is larger with the fcfs than with the market providers, while the opposite is true for large-scale farmers, as shown in Figure 6 panel (b). Service finding rates are defined analogously to findings rates in the search literature (Barnichon and Figura, 2015), i.e. the ratio between the number of serviced orders per period and the number of farms searching for a service, $\frac{q_{ij}\Delta_{ij}H}{F} = \frac{q_{ij}\Delta_{it}}{f}$.

The level of the rental rates are higher for small-scale than for large-scale farmers due to the higher cost of service per machine-hour rented (e.g. travel costs), see Table 10 panel (b) in Appendix D. At the same time, rental rates weigh differences in the probability of service across providers, with lower rental rates for the fcfs provider irrespective of scale. This is a consequence of lower conditional probabilities of service with the fcfs on average,

Figure 7: Relative Surplus



Notes: Expected surplus from a small-scale transaction (dark red diamonds) and a large-scale transaction (red dots) across providers, i.e. $\bar{V}_{ij} + U_i$. Ratio of expected surplus from the market to the fcfs provider. Hubs are ordered by increasing service capacity as in Table 4.

Δ_{ij} , particularly for large-scale farmers. The flip side of this feature are higher queues of small farmers with the fcfs and higher queues of large-scale farmers with the market provider, see Table 10 panel (a) in Appendix D. It is important to highlight that despite the fcfs has higher service probability for small-scale farmers, the conditional probabilities of service are lower than with the market providers. In other words, a higher service probability is driven by queue lengths. Figure 18 in Appendix D decomposes service finding rates into differences in queue lengths and service probabilities and compares them across providers.

Before describing the implications of adding further heterogeneity into this problem, we compute the expected surplus generated across providers service type, i.e. the sum of the expected profitability of providers and the value of market participation for servicing a farmers of a given scale, $\bar{V}_{ij} + U_i$. Figure 7 presents the size of the surplus for a transaction serviced by the mkt relative to the fcfs provider conditional on farmers' sizes. Two patterns emerge, the surplus generated by market providers is always larger than the one generated by fcfs. This should not be surprising since market providers could behave as fcfs providers if they wish to do so. More interestingly, the surplus from servicing large-scale farmers is higher than from servicing small-holder farmers.

5.3 Accommodating empirically relevant heterogeneity

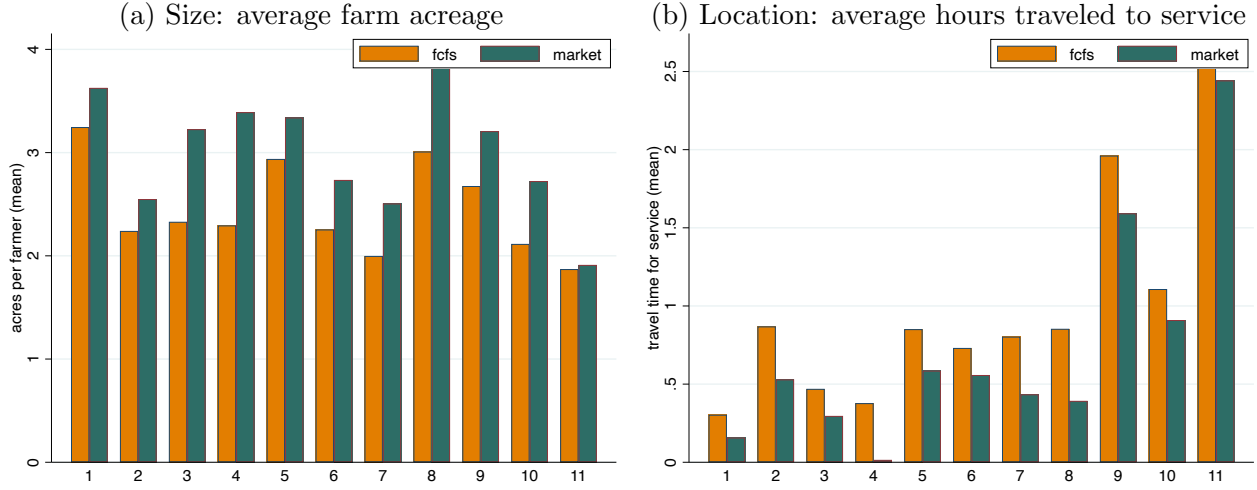
The stylized equilibrium queueing model does not capture for full extent of the observed heterogeneity in location and machine-hours demand. We accommodate this heterogeneity through simulation exercises where service queues to each provider are drawn from the empirical joint distribution of location and machine-hours observed in the catchment area of each hub. In other words, the queue lengths of small and large scale orders are the equilibrium ones, but their composition is allowed to vary following the empirical distribution of machine-hours and location observed in the data. We sample, with replacement, 1000 queues per provider. Each order in the queue is a three dimensional object that includes the machine-hours demanded, the location of the plot, and the productivity of the farmer that requested service. This productivity level (output per acre) is then used to compute the cost of equilibrium delays in service provision.²⁵

On the supply side, we solve the service dispatch system through two possible delivery routes. One follows a “hub and spoke” pattern, under which the equipment must return to the CHC between two consecutive orders. The other allows for a solution to a “Traveling Salesman Problem (TSP)”, where the implement travels optimally from order to order within the day. Under fcfs, the provider follows the route that minimizes the travel time within a given day for a given set of requests (and their order) in the queue. Under the market allocation, the type of requests being served are jointly determined with the best service route. The value of an order in the market allocation depends on the density of orders around them, and the size of the order relative to serving capacity.²⁶ We then estimate the value function for each provider, i.e. a function that maps any queue of orders to their service value, conditional on the dispatch system and the delivery route. As expected the value of service when we solve the TSP problem is always higher than without it, at the same time, the value of service provision is higher for market providers, and particularly so, for relatively close and larger orders, see Figure 19.

²⁵As robustness, we simulate outcomes when we assume no correlation between farm productivity and order sizes within the catchment area of a hub. These results are available upon request.

²⁶As we explain in Appendix C, this is a high dimensional problem, and the number of possible combinations of orders to be served within a period grows exponentially with the number of orders in the queue and its characteristics (including hours serviced and location, i.e., latitude and longitude).

Figure 8: Demand characteristics by provider



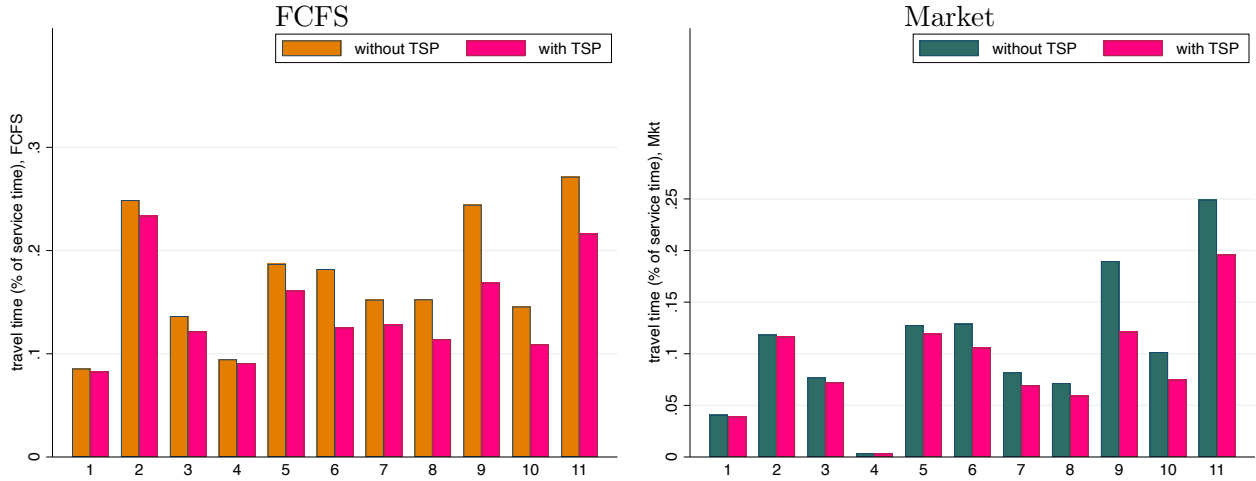
Notes: Panel (a) shows the sorting of farmers across providers by average farm acreage per order and Panel (b) shows the travel time to the service location, in hours. Hubs are ordered by increasing service capacity as in Table 4.

5.3.1 Farmer allocation across providers

We start by describing the sorting of farmers into different providers, classifying farmers by order size, i.e. the acreage of the plot for service; and by location, i.e. the travel time for service, see Figure 8.

The average order size served by a market provider is 3 acres while the average order size served by a fchs provider is 2.2 acres. This differential is a consequence of the disparities in the queue composition discussed before. At the same time, there are systematic differences in the travel time to locations. During a service day, market providers travel on average .8 hours (48 minutes), while fchs providers travel twice as much. This differential travel time reflects not only differences in the queue composition along the location dimension, but also disparities in the ability to prioritize orders. While equal-access-concerns may favour a fchs service arrangement, it is possible to improve upon the baseline allocation by allowing government subsidized hubs to optimize service delivery within a day. Figure 9 displays travel times when providers are allowed to solve a TSP among the orders served within each day. This option is particularly beneficial to fchs providers that are not allowed to prioritize order sizes. The average travel time (as % of service time) is 11% for the market provider and it declines to 9% once optimizing routes. The average travel time (as % of service time)

Figure 9: Travel times



Notes: Travel time by provider and dispatch system, “with TSP” correspond to allocations where providers are allowed to solve a traveling salesman problem for the orders serviced within a day. Travel time is expressed as a ratio of the total service within a day. Hubs are ordered by increasing service capacity as in Table 4.

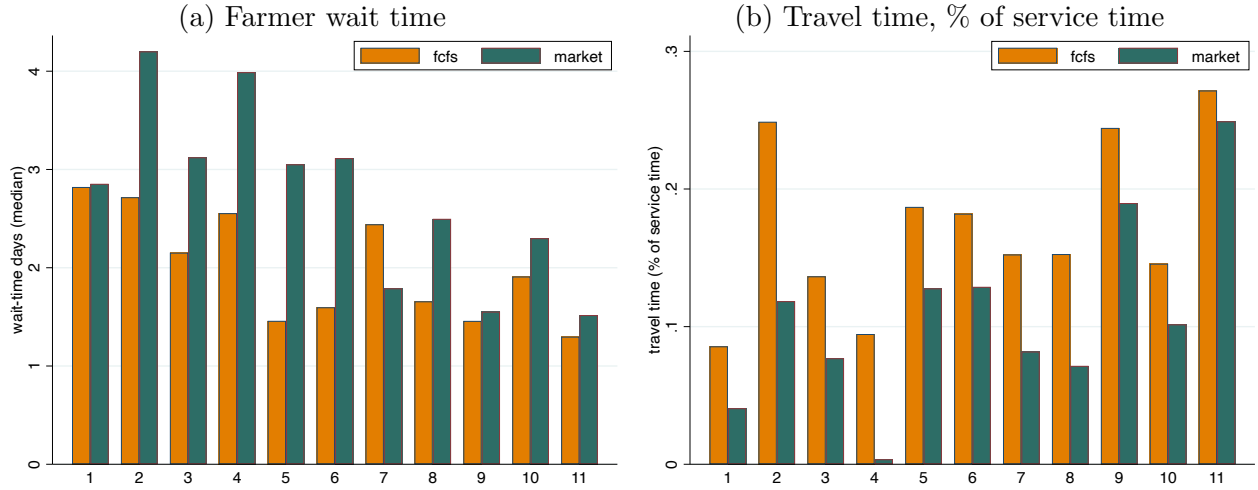
is 15% for the fcfs provider and it declines to 11.5% once solving the TSP.

5.3.2 Effectiveness in service delivery

Figure 10 panel (a) compares waiting times across the bootstrapped samples for different dispatch system and demand characteristics. On average, the mean wait time faced by farmers queueing with market providers is longer than that with fcfs. This feature is in part due to differences in the composition of the queue. Market providers have higher service rates for large-scale farmers, but those farmers are slightly more than 30% of the population of farmers in the economy on average. For the remainder of the farmers, market providers have lower service rates than fcfs providers (Figure 6), which is reflected in higher waiting time among those queueing with market providers.

Despite the better waiting time for farmers queueing with fcfs providers, these providers face equipment transportation costs (in terms of the opportunity cost of time) that can double those of the market providers. In other words, their inability to prioritize order sizes also affects their ability to service demand in space, traveling “too much” relative to their market counterparts, see Figure 10 panel (b). It is not surprising then that the value of

Figure 10: Time management by hub



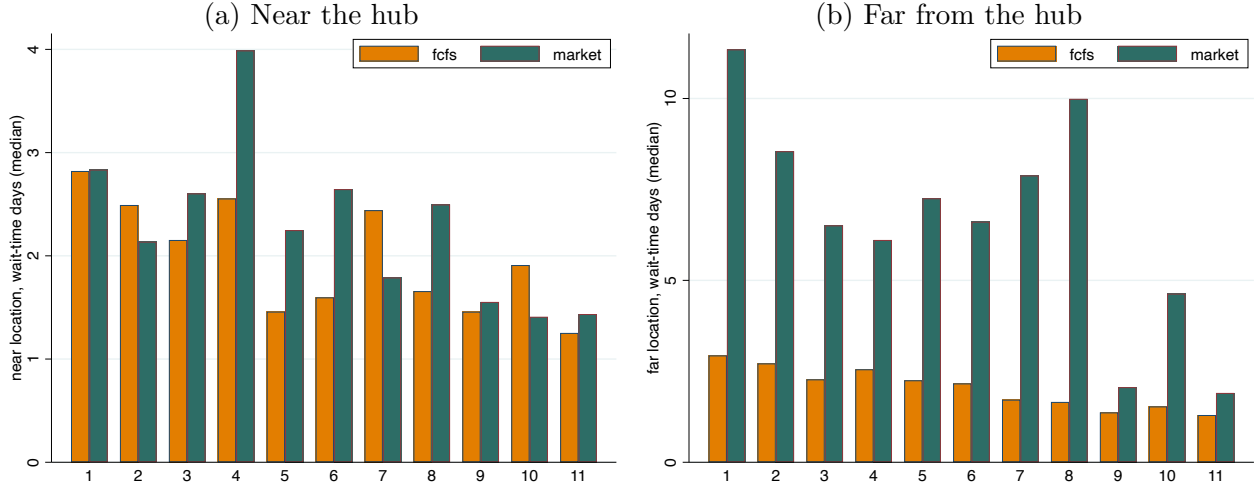
Notes: Panel (a) shows average days waiting for service across providers for the average farm in the catchment area of each hub. Panel (b) shows the average travel time as a ratio of the service time by provider and hub. Hubs are ordered by increasing service capacity as in Table 4.

optimizing service routes is higher for fcs providers as we showed before. Finally, notice that hubs are ordered in terms of service capacity with hub 1 holding less than a third of the service capacity of hub 11. On average, travel time as % of service time increases as service capacity increases, and this correlation is stronger for the market provider than it is for the fcs provider.

5.3.3 The cost of delays and of service provision in space

Delays in service provision are costly for farmers. Delays are an equilibrium outcome in our economy given the nature of search in the market for rental equipments, the disparities in service provision across providers and the spatial distribution of service demand. Figure 11 displays the distribution of waiting time for service time across farmers located near the catchment area of a hub (less than 30 minutes of travel time, left panel) and those located far (more than 30 minutes of travel time, right panel). Most of the excess delay in near locations observed for market providers has to do with the sorting of demand by size, i.e. the market providers attract relatively more large-scale farmers, which induces delays in service provision toward smaller farmers. The difference in delays between providers is maximized for far away locations, because market providers systematically avoid servicing

Figure 11: Delays in space

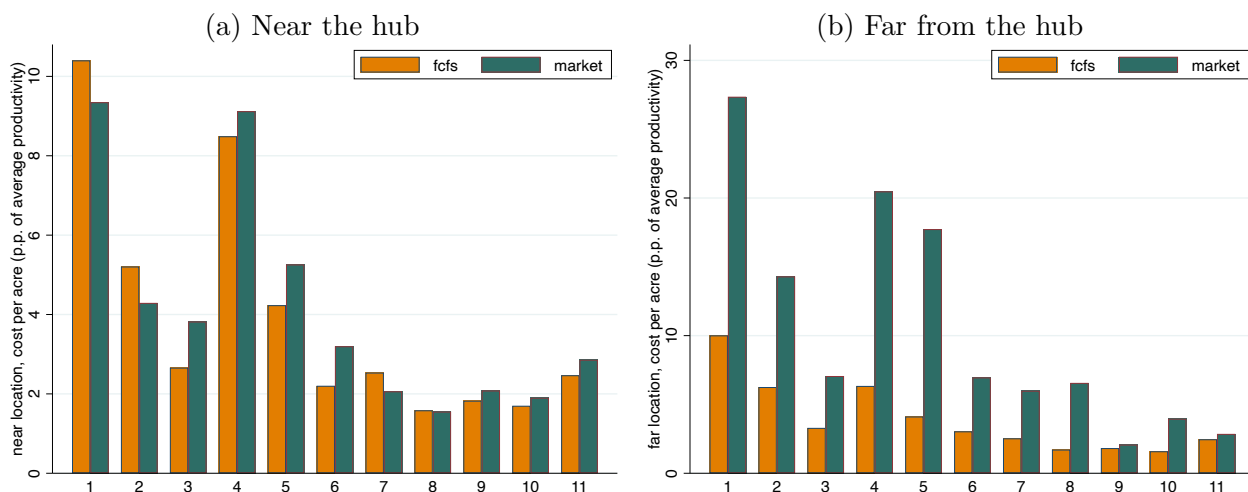


Notes: Panel (a) shows the average days waiting for service across providers for the average farm in locations within 30 minutes of travel of the centroid of each hub. Panel (b) shows the same measure for the average farm in locations more than 30 minutes away from the centroid of each hub. Hubs are ordered by increasing service capacity as in Table 4.

those.

These delays induce costs in revenue per acre that depend on the joint distribution of location, size and productivity. Figure 12 reports the productivity cost per acre across providers for the bootstrapped samples. These are measured as the decline in revenue per acre relative to the average revenue per acre in the catchment area of a hub. Despite the documented disparities in delays across providers in near locations, productivity costs per acre are similar across dispatch systems. For example, in hub 4, the market provider induces a delay in provision of roughly an additional day relative to the fchs provider (from 2.5 to 4 days). However, those farmers that wait longer are on average less productive and therefore the cost per acre is similar across providers, 8.4% of average revenue per acre for the fchs provider and 9% for the market provider. Conditional on servicing far away locations, the disparities in delays are so large across providers, that those delays induce stronger productivity costs for those queueing with the market providers. Still, as in nearby locations, the disparities in productivity are smaller than in delays across providers.

Figure 12: Productivity costs in space



Notes: Panel (a) shows the cost in revenue per acre for the average farm in locations within 30 minutes of travel of the centroid of each hub. Panel (b) shows the same measure for the average farm in locations more than 30 minutes away from the centroid of each hub. The cost is measured in p.p. of the average revenue per acre in the catchment area of each hub. Hubs are ordered by increasing service capacity as in Table 4.

6 The value of the subsidies

6.1 The market prior to the subsidy

We now study how the status quo equilibrium compares to the equilibrium prior to the intervention. While data for the pre-subsidy market is unavailable, we can account for this effect by running a counterfactual analysis where we shut down the supply of equipment stemming from fcs, i.e. the government subsidized supply. This counterfactual is valid under the assumption that market participants accommodated the entry of new supply in the market. This is a plausible scenario due to the low ownership of equipment in the population and the desire of most farmers to engage in rentals.

The substantial increase in equipment supply due to the subsidy implies an increase in the service finding rates, moving from 15% prior to the subsidy to between 40% and 55% after the subsidy depending on the provider, see Table 6. Gains in service findings rates are mostly attributed to small farmers, which prior to the subsidy faced a service finding rate of 6% and after the subsidy face a service finding rate of 56%, with a bit more than half of it attributed to the fcs providers. Market providers' service probability also increases in

equilibrium because the cost of service declines in response to stronger competition. Rental rates fall by 18% for small scale farmers and by 5% for large farmers. The differential effect is a consequence of the implicit priority given to small-holder farmers by the fcfs subsidized provider.

Table 6: Effect of the subsidy

	prior to policy	post subsidy	
		fcfs	mkt
Service Finding Rate			
average farmer	0.15	0.41	0.56
small-scale	0.05	0.28	0.22
large-scale	0.1	0.13	0.34
Rental Rate			
small-scale	142	121	125
large-scale	94	93	94

Notes: This table shows the service findings rates and hourly rental rates (in rupees) across different equilibrium allocations. These results are presented by farmer scale of production and by provider. The “prior to policy” equilibrium corresponds to an allocation where only the market suppliers are available. The “post subsidy” equilibrium corresponds to our benchmark economy.

6.2 Market Deregulation

One of the findings in Section 5.3.2 is that fcfs providers could benefit from operating a technology that allows them to optimize equipment in space as well as optimize the type of orders being served. We study the effect of a market deregulation through counterfactuals. Because prioritizing large-scale farmers is costless and the fcfs providers are at least as well off as before (i.e. they can now prioritize the high marginal return orders), a profit driven fcfs provider would choose to adopt the technology, leading to a shift in provider composition towards $h = 0$.²⁷ In other words, there is no longer any differentiation between these two types of providers. The nature of the equilibrium may however change due to the endogenous farmer sorting, and pricing of services.

The short run response of the economy (without endogenous entry or exit of providers) implies higher service finding rates for large-scale farmers relative to the fcfs provision, consistently with the profit maximizing strategy of market providers (see Table 7). However,

²⁷As we pointed out before, the value of service for market providers is always above the one for first-come-first-served providers, and therefore each provider has incentives to adopt a dispatch system that prioritizes large orders.

Table 7: Effect of Market Deregulation

	Benchmark		Deregulation	
	fcfs	mkt	short-run	long-run
Service Finding Rate				
average farmer	0.41	0.56	0.49	0.5
small-scale	0.28	0.22	0.29	0.3
large-scale	0.13	0.34	0.2	0.21
Rental Rate				
small-scale	121	125	122	120
large-scale	93	94	92	92

Notes: This table shows the service findings rates and hourly rental rates (in rupees) across different equilibrium allocations. These results are presented by farmer scale of production and by provider. The “benchmark allocation” corresponds to the calibrate economy, whereas the “deregulated” economy corresponds to economies where fcfs providers are allowed to prioritize orders as their market counterparts. The “short-run” is an equilibrium without providers’ entry-and-exit whereas the “long-run” allows for it.

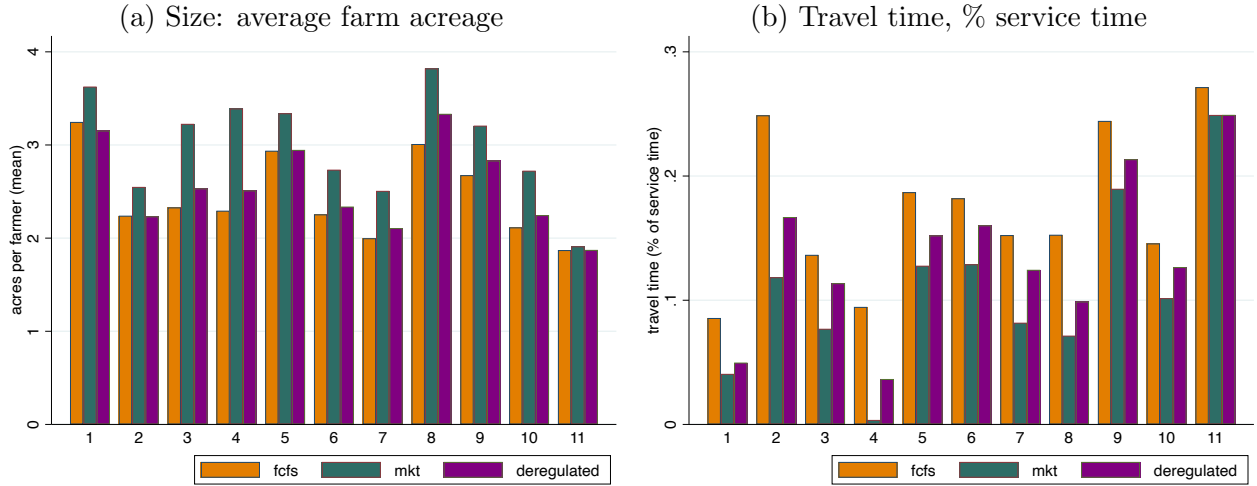
their service finding rates are below those of small scale farmers that are drawn to the market in response to the increased supply of services. While service findings rates for small-holder farmers do not change relative to the baseline, the rental costs increase for all farmers. The long run response of the economy (with endogenous entry and exit) restores rental costs to their baseline levels, and generates allocations where service findings rates are higher for small holder farmers than large holder farmers, despite market providers prioritizing the latter. The reason is that small-scale requests are valuable when there are service capacity constraints because they improve capacity utilization.

Figure 13 plots the change in average farm sizes served and the travel time for service, i.e. the change in the distribution of served location, as the market deregulates. Exit of providers induces an increase in the average size of the farm served by each provider, and a reduction in the travel time to service, consistently with providers prioritizing services with low marginal cost of provision. Travel time as a proportion of the service time declines by 20%, on average across hubs.

6.3 Small-holder access and equipment supply.

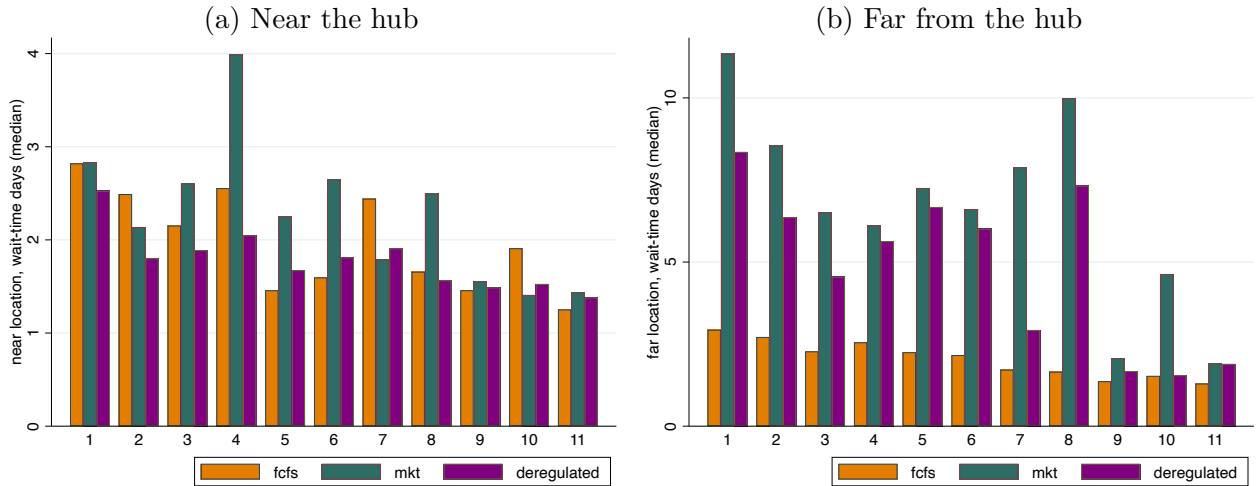
An extensive literature in agricultural economies has studied the effect of government subsidies for equipment. Two conclusions arise: (a) equipment subsidies are regressive because they benefit relatively wealthier farmers; (b) the impact of these subsidies in mechanization

Figure 13: Impact of Deregulation, demand characteristics



Notes: Panel (a) shows the sorting of farmers by average farm acreage per order. Panel (b) shows the average travel time to the service location as a ratio of the average service time. Hubs are ordered by increasing service capacity as in Table 4.

Figure 14: Impact of Deregulation, delays



Notes: Panel (a) shows the average days waiting for service across providers for the average farm in locations within 30 minutes of travel of the centroid of each hub. Panel (b) shows the same measure for the average farm in locations more than 30 minutes away from the centroid of each hub. In purple we include outcomes for the deregulated economy, i.e. where fcs providers can prioritize orders. Hubs are ordered by increasing service capacity as in Table 4.

is widely heterogeneous, with only a handful of successful cases (see review in [Pingali, 2007](#)).

Our results suggest a different interpretation of the heterogeneity in the effectiveness of subsidies. First, small-holder farmers can benefit disproportionately more from these subsidies than large-scale farmers (at least in terms of service findings rates), as they do in our benchmark result. Their ability to do so depends on their value in terms of capacity utilization, or the density of their location in space; and the generosity of the subsidy, or the increase in the supply of machine-hours. Indeed, if the supply increase is relatively low, service finding rates for small-holder farmers are below that of large-scale farmers as we show in [Table 8](#). One could erroneously interpret this results as the subsidy being regressive.

[Table 8](#) also highlights that heterogeneity in the success of these government programs for generating long-term shifts in mechanization may have been related to heterogeneity in the subsidy amounts, and the joint spatial and size distribution of farms.

Table 8: Service finding rates across supply of machine-hours, H .

	H_{ps}	H relative to H_{ps}			
	1	1.4	2	3.3	5
average farm	0.15	0.21	0.28	0.39	0.47
small-scale	0.05	0.08	0.12	0.19	0.26
large-scale	0.10	0.13	0.16	0.19	0.20

Note: Column H_{ps} corresponds to the level of pre-subsidy supply of machine-hours. The pre-subsidy supply level is estimated as the equipment supply absent CHC’s machine hours, see [Section 6.1](#). Service finding rates are computed as described in the text.

7 Conclusion

Rental markets hold considerable promise in expanding mechanization access and increasing productivity in the farming sector. However, the spatial distribution of demand in space and its synchronous nature, as well as the fixed supply capacity, poses important policy-relevant trade offs for service provision. The returns to these rental markets depend crucially on factors such as spatial density, i.e. the proximity of suppliers to farmers, the overall supply capacity, and the ability to optimize traveling equipment time. In this paper, we document and quantify how these factors determine the distributional effects of rental markets.

We find that when the government increases service capacity by subsidizing the purchase of equipment for rental service provision, and at the same time imposes a first-come-first-serve dispatch system to allocate services, it induces misallocation in service provision. When equipment owners are allowed to maximize their profits by prioritizing larger scale orders, the equilibrium allocation may induce higher finding rates for small-scale producers. The reason is that small-scale orders are valuable in terms of service capacity-utilization, particularly when located in high-density areas.

We provide a parsimonious framework to study the allocation of shared services across heterogenous demand and spatial allocations. The framework can be readily extended to study similar markets where capacity utilization and geography are important determinants in service provision. Finally, while we take the location of service providers as given, a natural step forward would be to study the properties of the endogenous location of providers in space, as in [Oberfield et al. \(2020\)](#).

References

- Adamopoulos, T. and Restuccia, D. (2014). The size distribution of farms and international productivity differences. *American Economic Review*, 104(6):1667–97.
- Adamopoulos, T. and Restuccia, D. (2021). Geography and Agricultural Productivity: Cross-Country Evidence from Micro Plot-Level Data. *The Review of Economic Studies*, 89(4):1629–1653.
- Barnichon, R. and Figura, A. (2015). Labor market heterogeneity and the aggregate matching function. *American Economic Journal: Macroeconomics*, 7(4):222–49.
- Bassi, V., Muoio, R., Porzio, T., Sen, R., and Tugume, E. (2022). Achieving scale collectively. *Econometrica*, 90(6):2937–2978.
- Brancaccio, G., Kalouptsi, M., and Papageorgiou, T. (2020). A guide to estimating matching functions in spatial models. *International Journal of Industrial Organization*, 70:102533.
- Burdett, K. and Judd, K. L. (1983). Equilibrium price dispersion. *Econometrica*, 51(4):955–969.
- Burdett, K., Shi, S., and Wright, R. (2001). Pricing and matching with frictions. *Journal of Political Economy*, 109(5):1060–1085.
- Caunedo, J. and Kala, N. (2021). Mechanizing agriculture. Working Paper 29061, National Bureau of Economic Research.
- Caunedo, J. and Keller, E. (2020). Capital Obsolescence and Agricultural Productivity*. *The Quarterly Journal of Economics*, 136(1):505–561.
- Chen, C. (2020). Technology adoption, capital deepening, and international productivity differences. *Journal of Development Economics*, 143:102388.
- Daum, T., Villalba, R., Anidi, O., Mayienga, S. M., Gupta, S., and Birner, R. (2021). Uber for tractors? opportunities and challenges of digital tools for tractor hire in india and nigeria. *World Development*, 144:105480.
- Duranton, G. and Puga, D. (2004). Chapter 48 - micro-foundations of urban agglomeration economies. In Henderson, J. V. and Thisse, J.-F., editors, *Cities and Geography*, volume 4 of *Handbook of Regional and Urban Economics*, pages 2063–2117. Elsevier.
- Foster, A. D. and Rosenzweig, M. R. (2022). Are there too many farms in the world? labor market transaction costs, machine capacities, and optimal farm size. *Journal of Political Economy*, 130(3):636–680.
- Frechette, G. R., Lizzeri, A., and Salz, T. (2019). Frictions in a competitive, regulated market: Evidence from taxis. *American Economic Review*, 109(8):2954–92.
- Holmes, T. J. and Lee, S. (2012). Economies of Density versus Natural Advantage: Crop

- Choice on the Back Forty. *The Review of Economics and Statistics*, 94(1):1–19.
- Judd, K. L., Maliar, L., Maliar, S., and Valero, R. (2014). Smolyak method for solving dynamic economic models: Lagrange interpolation, anisotropic grid and adaptive domain. *Journal of Economic Dynamics and Control*, 44(C):92–123.
- Kala, N. (2017). Learning, adaptation, and climate uncertainty: Evidence from indian agriculture. *MIT Center for Energy and Environmental Policy Research wp*, 23.
- Lagos, R. (2000). An alternative approach to search frictions. *Journal of Political Economy*, 108(5):851–873.
- Manuelli, R. and Seshadri, A. (2014). Frictionless Technology Diffusion: The Case of Tractors. *American Economic Review*, 104(4):1368–91.
- Marshall (1890). *Principles of Economics*. London: Macmillan.
- Oberfield, E., Rossi-Hansberg, E., Sarte, P.-D., and Trachter, N. (2020). Plants in space. Working Paper 27303, National Bureau of Economic Research.
- Pingali, P. (2007). *Agricultural Mechanization: Adoption Patterns and Economic Impact*, volume 3 of *Handbook of Agricultural Economics*, chapter 54, pages 2779–2805. Elsevier.
- Pingali, P., Bigot, I., and Binswanger, H. P. (1988). Agricultural mechanization and the evolution of farming systems in sub-saharan africa. Technical report, The World Bank.
- Rosenthal, R. W. (1980). A model in which an increase in the number of sellers leads to a higher price. *Econometrica*, 48(6):1575–1579.
- Rossi-Hansberg, E. (2005). A spatial theory of trade. *American Economic Review*, 95(5):1464–1491.
- Sattinger, M. (2002). A queuing model of the market for access to trading partners. *International Economic Review*, 43(2):533–547.
- Satyasai, K. and Balanarayana, M. (2018). Can mechanization in agriculture help achieving sustainable development goals? *Agricultural Economics Research Review*, 31(347-2018-5155):147–156.
- Shi, S. (2002). A directed search model of inequality with heterogeneous skills and skill-biased technology. *The Review of Economic Studies*, 69(2):467–491.
- Suri, T. and Udry, C. (2022). Agricultural technology in africa. *Journal of Economic Perspectives*, 36(1):33–56.
- Varian, H. R. (1980). A model of sales. *The American Economic Review*, 70(4):651–659.
- Wright, R., Kircher, P., Julien, B., and Guerrieri, V. (2021). Directed search and competitive search equilibrium: A guided tour. *Journal of Economic Literature*, 59(1):90–148.
- Yamauchi, F. (2016). Rising real wages, mechanization and growing advantage of large farms: Evidence from Indonesia. *Food Policy*, 58(C):62–69.
- Yang, J., Huang, Z., Zhang, X., and Reardon, T. (2013). The rapid rise of cross-regional

agricultural mechanization services in china. *American Journal of Agricultural Economics*, 95(5):1245–1251.

A Characterization of the probability of service.

A.1 Probability of service for a particular farmer

The probability that a provider chooses a particular farmer given that he chooses one of type i is $\tilde{\Delta}_{ij}(1)$, i.e.

$$\tilde{\Delta}_{ij}(1) = \sum_{n=0}^{f_i-1} \binom{f_i-1}{n} p_{ij}^n (1-p_{ij})^{f_i-1-n} \frac{1}{n+1},$$

where f_i is the number of farmers of type i searching for a provider, $f_s = sF$ and $f_{s-} = (1-s)F$ and $\binom{f_i-1}{n} = \frac{(f_i-1)!}{n!(f_i-1-n)!}$. Hence,

$$\tilde{\Delta}_{ij}(1) = \frac{1 - (1-p_{ij})^{f_i}}{f_i p_{ij}}.$$

As the number of agents in the economy gets large, and using the definition of queue lengths above, the service probability simplifies to

$$\tilde{\Delta}_{ij}(1) = \frac{1 - e^{-q_{ij}}}{q_{ij}}.$$

That is, the probability that at least one farmer of type i has requested a service, $1 - e^{-q_{ij}}$, divided by the number of requests of a given type, q_{ij} .

Next, consider the probability of a particular farmer being served when the provider serves $\bar{o} = 2$ orders of type i , $\tilde{\Delta}_{ij}(2)$. Similar computations to those above yield a service probability as follows

$$\tilde{\Delta}_{ij}(2) = 2\left(\frac{1 - e^{-q_{ij}}}{q_{ij}}\right) - e^{-q_{ij}}.$$

Finally, consider the probability of a particular farmer being served when the provider serves $\bar{o} = 3$ orders of type i , $\tilde{\Delta}_{ij}(3)$, which follows

$$\tilde{\Delta}_{ij}(3) = 3\left(\frac{1 - e^{-q_{ij}}}{q_{ij}}\right) - 2e^{-q_{ij}} - e^{-q_{ij}} q_{ij}.$$

A.2 Probability of service of a farmer of type i

Next, we characterize the probability that a provider of type j services a farmer of type i given that a farmer of type i is standing in the queue.

First-come-first-served. The fcfs provider only considers feasibility and the position of the farmer in the queue. Let the probability of serving \bar{o} farmers of type i be $\phi_{i,fcfs}(\bar{o})$.

Given the queue lengths at this provider, there are $q_s+q_s-P_o = \frac{q_s+q_s-!}{(q_s+q_s--o)!}$ possible permutations for the o-tuple, (the provider identifier has been dropped for notational convenience). Under Assumption 1, a fcfs provider serves a single large-scale farmer if one of the large-scale farmers are among the first three positions in the queue, which occurs with probability $\hat{\phi}_{s,fcfs}(1) \equiv 3q_s \frac{q_s-P_2}{q_s+q_s-P_3}$; and at least one has applied.

$$\phi_{s,fcfs}(1) = \psi_{s,fcfs}(1)\hat{\phi}_{s,fcfs}(1),^{28}$$

where $\psi_{s,fcfs}(1) \equiv (1 - e^{-q_{s^-,fcfs}} - q_{s^-,fcfs}e^{-q_{s^-,fcfs}})$ is the probability of having at least three orders in the queue of which at least two are of type s^- , when a single farmer of type s has requested service. To this probability we should add the probability of service when less than 3 farmers apply for service. The latter is $\hat{\psi}_{s,fcfs}(1) \equiv (e^{-q_{s,fcfs}}(e^{-q_{s^-,fcfs}} + q_{s^-,fcfs}e^{-q_{s^-,fcfs}}))$, i.e. the probability of service of large scale order when there are no other service request or there is exactly one additional order requested.

A fcfs provider services 2 large-scale farmers if there are two or more large-scale orders in the first o positions of the queue and at least two large scale farmers have requested service. Let the first probability be $\hat{\phi}_{s,fcfs}(2) \equiv 3q_s(q_s - 1) \frac{q_s-2+q_s-P_1}{q_s+q_s-P_3}$

$$\phi_{s,fcfs}(2) = \psi_{s,fcfs}(2)\hat{\phi}_{s,fcfs}(2),$$

where $\psi_{s,fcfs}(2) = (1 - e^{-q_{s,fcfs}} - e^{-q_{s^-,fcfs}}q_s e^{-q_{s,fcfs}})$ is the probability that there are at least three orders in the queue conditional of a farmer of type s requesting service, of which at least two are of type s (including the one requesting service).²⁹ To this probability we should add the probability that there are only two large-scale farmers in the queue $\hat{\psi}_{s,fcfs}(2) \equiv (q_{s,fcfs}e^{-q_{s,fcfs}}e^{-q_{s^-,fcfs}})$.

Given feasibility, the fcfs provider never serves 3 large-scale orders, $\phi_{s,fcfs}(3) = 0$.

A fcfs provider serves a single small-holder farmer if there is one of them in the first o positions of the queue. This probability is defined analogously to its counterpart for large scale orders,

$$\phi_{s^-,fcfs}(1) = \psi_{s^-,fcfs}(1)\hat{\phi}_{s^-,fcfs}(1),$$

and adding the probability $\hat{\psi}_{s^-,fcfs}(1)$ when there are less than three orders.

A fcfs provider services 2 small-holder farmers if at least two small-scale orders in the

²⁸Note that $\hat{\phi}_{s,fcfs}(i)$ are not the expected probabilities, but rather the probability conditional on the observed queue length. We can numerically show that when $F, H \rightarrow \infty$ these two are arbitrarily close.

²⁹This is the probability that at least another large scale and at least one small scale farmer request service, or at least two other large scale farmers request service.

first o positions of the queue, $\hat{\phi}_{s^-,fcfs}(2) \equiv 3 \frac{q_{s^-}(q_{s^-}-1)q_s}{q_s+q_{s^-}-P_3}$

$$\phi_{s^-,fcfs}(2) = \psi_{s^-,fcfs}(2)\hat{\phi}_{s^-,fcfs}(2),$$

where $\psi_{s^-,fcfs}(2) = (1 - e^{-q_{s^-,fcfs}})(1 - e^{-q_{s^-,fcfs}})$ is the probability that there are at least three orders in the queue conditional of a farmer of type s^- requesting service, of which at least one is of type s and at least two are of type s^- (including the one requesting the service). To this probability we should add the probability that there are only two orders in the queue, $\hat{\psi}_{s^-,fcfs}(2)$ defined analogously than for large-scale farmers.

A fcfs provider services 3 small-holder farmers if there are three small-scale orders in the first o positions of the queue. This probability is defined as $\hat{\phi}_{s^-,fcfs}(3) = \frac{q_{s^-}P_3}{q_s+q_{s^-}-P_3}$

$$\phi_{s^-,fcfs}(3) = \psi_{s^-,fcfs}(3)\hat{\phi}_{s^-,fcfs}(3),$$

where $\psi_{s^-,fcfs}(3)$ is the probability of having at least two other small scale requests, i.e. $\psi_{s^-,fcfs}(3) = (1 - e^{-q_{s^-,fcfs}} - q_{s^-,fcfs}e^{-q_{s^-,fcfs}})$.

The general form for the probability of service is,

$$\Delta_{i,fcfs} = \sum_{\bar{o}=1}^3 \hat{\psi}_{i,fcfs}(\bar{o}) + \phi_{i,fcfs}(\bar{o})\tilde{\Delta}_{i,fcfs}(\bar{o}), \quad (10)$$

where we have defined $\hat{\psi}_{s,fcfs}(3) \equiv 0$ to ease notation.

The main difference in the probability of service for large and small relies on the queue lengths. If the queue lengths are identical, then a first-come-first-served provider serves both types of farmers with the same probability, $\sum_{\bar{o}=2}^3 \phi_{s^-,fcfs}(\bar{o}) = \sum_{\bar{o}=2}^3 \phi_{s,fcfs}(\bar{o})$.

Market. The market provider has a technology that allows him to prioritize farmers of either type. The probability of interest is the probability that exactly \bar{o} farmers of type i are served conditional on the farmer under consideration having applied.

Conditional on a large farmer having applied, a single large-scale farmer is served by a market provider if the provider does not prioritize large scale farmers and there is one large-scale order among the first o available positions, which happens with probability $(1 - \chi)\tilde{\phi}_{s,mkt}(1) = (1 - \chi)\phi_{s,fcfs}(1)$; or if the provider prioritizes large scale farmers and no other large-scale farmer requested service, $\chi\psi_{s,mkt}(1) = \chi e^{-q_{s,mkt}}(1 - e^{-q_{s^-,mkt}} - q_{s^-,mkt}e^{-q_{s^-,mkt}})$. These service probabilities add up to,

$$\phi_{s,mkt}(1) = \chi(\psi_{s,mkt}(1)) + (1 - \chi)\tilde{\phi}_{s,mkt}(1),$$

the latter assumes that at least three orders have been requested. We should add to these the event that there are less than three orders, which happens with probability $\hat{\psi}_{i,mkt}(1) = \hat{\psi}_{i,fcfs}(1)$ for any $i=s, s^-$ by definition.

Two large-scale farmers are served by a market provider if he does not prioritize large orders and they stand in the first 3 positions, which happens with probability $(1-\chi)\tilde{\phi}_{s,mkt}(2) = (1-\chi)\phi_{s,fcfs}(2)$; or if the provider prioritizes those orders and there is at least one additional large-scale service request, which happens with probability $\chi\psi_{s,mkt}(2) = \chi(1 - e^{-q_{s,mkt}} - e^{-q_{s^-,mkt}}q_s e^{-q_{s,mkt}})$.³⁰ These service probabilities add up to

$$\phi_{s,mkt}(2) = \chi(\psi_{s,mkt}(2)) + (1 - \chi)\tilde{\phi}_{s,mkt}(2).$$

We then add the probability when less than three orders are in the queue, $\hat{\psi}_{s,mkt}(2)$ defined analogously to the fcfs provider.

Feasibility prevents three large-scale orders to be served within the period and therefore, $\phi_{s,mkt}(3) = 0$.

Analogous arguments can be used to describe the probabilities of service of small scale farmers. A single small-holder farmer is always served by a market provider (conditional on a request) if it prioritizes high-scale requests and at least two large scale farmers have requested service, which occurs with probability $\chi\psi_{s^-,mkt}(1) = \chi(1 - e^{-q_{s,mkt}} - q_{s,mkt}e^{-q_{s,mkt}})$; or if the provider does not prioritize high-scale requests and there is a single small-scale order among the first three orders in the queue, $(1-\chi)\tilde{\phi}_{s^-,mkt}(1)$, where $\tilde{\phi}_{s^-,mkt}(1) = \phi_{s^-,fcfs}(1)$. The reason for always serving a small scale order even when prioritizing large scale is that capacity constraints allow the provider to serve at most $o-1$ orders leaving always an idle slot. Finally, we add the probability that there are less than two large-scale orders in the queue.

$$\phi_{s^-,mkt}(1) = \chi(\psi_{s^-,mkt}(1)) + (1 - \chi)\tilde{\phi}_{s^-,mkt}(1),$$

and $\hat{\psi}_{s^-,mkt}(1)$.

Two small-holder farmers are served by a market provider if it prioritizes high-scale requests and exactly one large-scale farmer requests service and at least another small scale farmer requests service, which occurs with probability $\chi\psi_{s^-,mkt}(2) = \chi q_{s,mkt} e^{-q_{s,mkt}} (1 - e^{-q_{s^-,mkt}})$. Alternatively, two small-holder farmers are served if the provider does not prioritize large-scale orders and there are two small-scale orders among the first three orders in

³⁰If there are more large-scale orders the provider still serves two because of its capacity constraints.

the queue.

$$\phi_{s^-,mkt}(2) = \chi(\psi_{s^-,mkt}(2)) + (1 - \chi)\tilde{\phi}_{s^-,mkt}(2).$$

To these probabilities we add those associated to the event when there are strictly less than two orders in the queue, $\hat{\psi}_{s^-,mkt}(2)$.

Three small-holder farmers are served by the market provider if it prioritizes high-scale requests and no large-scale farmer requests service and there are at least three small requests, which occurs with probability $\chi\psi_{s^-,mkt}(3) = \chi e^{-q_{s^-,mkt}}(1 - e^{-q_{s^-,mkt}} - q_{s^-,mkt}e^{-q_{s^-,mkt}})$, or if it does not prioritize them and there are three small-scale orders among the first three in the queue,

$$\phi_{s^-,mkt}(3) = \chi\psi_{s^-,mkt}(3) + (1 - \chi)\tilde{\phi}_{s^-,mkt}(3).$$

In sum, the probability of service for a market provider follows

$$\Delta_{i,mkt} = \sum_{\bar{o}=1}^3 \hat{\psi}_{i,mkt}(\bar{o}) + \phi_{i,mkt}(\bar{o})\tilde{\Delta}_{i,mkt}(\bar{o}). \quad (11)$$

Following equations 10 and 11, the probability of being served for a given type i (weakly) declines in the queue length of the other type of farmers. In the first-come-first-served provider the result is straightforward. For the market provider, the decline in the probability of service is strict for the small scale farmers and independent of the queue length of small-scale orders when the provider prioritizes large-scale orders.

A.3 Unconditional probabilities of service.

The unconditional probabilities of service are important in characterizing the value of service for each provider. We consider alternative scenarios, i.e. when the provider serves at capacity ($o = 3$ orders) and when the provider serves less than capacity.

The first-come-first-served provider can serve three orders of small scale (given feasibility) with a service probability of

$$\Phi_{s^-,fcfs}(3) = \left(1 - e^{-q_{s^-,j}}(1 + q_{s^-,j} + \frac{1}{2}q_{s^-,j}^2)\right) \frac{q_{s^-,j}P_3}{q_{s^-,j} + q_{s^-,j}P_3};$$

or to serve two orders of one type and one of another, with probability

$$\bar{\Phi}_{i,fcfs}(1) = (1 - e^{-q_{i,j}})(1 - e^{-q_{i',j}} - q_{i',j}e^{-q_{i',j}}) \frac{3q_{i,j}q_{i',j}(q_{i',j} - 1)}{q_{i,j} + q_{i',j}P_3},$$

and $\bar{\Phi}_{i',fcfs}(2) = \bar{\Phi}_{i,fcfs}(1)$ for $i' \neq i$.

The provider can also serve two orders of large size, (either because he received only two orders, or because all orders in the queue are of large scale)

$$\tilde{\Phi}_{s,fcfs}(2) = (1 - e^{-q_{s,fcfs}} - q_{s,fcfs}e^{-q_{s,fcfs}}) e^{-q_{s-,fcfs}},$$

or it can receive exactly two orders of small size and serve those,

$$\tilde{\Phi}_{s-,fcfs}(2) = \frac{1}{2}q_{s-,j}^2 e^{-q_{s-,j}} (e^{-q_{s,j}}).$$

Finally, the provider can serve two orders, one of each type

$$\tilde{\Phi}_{i,fcfs}(1_2) = (q_{i,j}e^{-q_{i,j}} q_{i',j}e^{-q_{i',j}}).$$

or only one order, with occurs with probability

$$\tilde{\Phi}_{i,fcfs}(1_1) = (q_{i,j}e^{-q_{i,j}} e^{-q_{i',j}}).$$

The probabilities for the market provider are similar to the ones above, except that we need to account for the market provider's ability to select large scale orders.

The market provider can serve three orders of small scale,

$$\bar{\Phi}_{s-,mkt}(3) = \chi e^{-q_{s,mkt}} (1 - e^{-q_{s-,mkt}} - q_{s-,mkt}e^{-q_{s-,mkt}} - \frac{1}{2}q_{s-,mkt}^2 e^{-q_{s-,mkt}}) + (1 - \chi)\bar{\Phi}_{s-,fcfs}(3), \quad (12)$$

or the orders of large scale and one small,

$$\bar{\Phi}_{s,mkt}(2) = \chi((1 - e^{-q_{s,mkt}} - q_{s,mkt}e^{-q_{s,mkt}})(1 - e^{-q_{s-,mkt}}) + (1 - \chi)\bar{\Phi}_{s,fcfs}(2)), \quad (13)$$

or two orders of small scale and on large,

$$\bar{\Phi}_{s-,mkt}(2) = \chi q_{s,mkt} e^{-q_{s,mkt}} (1 - e^{-q_{s-,mkt}} - q_{s-,mkt}e^{-q_{s-,mkt}}) + (1 - \chi)\bar{\Phi}_{s-,fcfs}(2). \quad (14)$$

When there are less than three orders in the queue there is no need to prioritize orders, and therefore the probabilities of service are identical to those characterized for the FCFS problem, i.e. $\tilde{\Phi}_{i,mkt} = \tilde{\Phi}_{i,fcfs}$.

Expected value of service provision The characterization of the probability allows

us to compute the expected value of service provision:

$$\begin{aligned}
& \tilde{V}(\{\bar{o}_s, \bar{o}_{s^-}\}_{\mathbf{q}_{\text{fcfs}}}, \{(r_{i,\text{fcfs}} - w)k_i - wd_i\}_{i=s, s^-}) \equiv \\
& \sum_{i=s, s^-} \bar{\Phi}_{i,\text{fcfs}}(2) \left[2((r_{i,\text{fcfs}} - w)k_i - wd_i) + (r_{i',\text{fcfs}} - w)k_{i'} - wE(d_{i'}) \right] + \\
& \quad \tilde{\Phi}_{i,\text{fcfs}}(1_1) ((r_{i,\text{fcfs}} - w)k_i - wd_i) + \\
& \quad + \tilde{\Phi}_{i,\text{fcfs}}(1_2) ((r_{i,\text{fcfs}} - w)k_i - wd_i + (r_{i',\text{fcfs}} - w)k_{i'} - wE(d_{i'})) + \\
& \quad \tilde{\Phi}_{i,\text{fcfs}}(2) 2((r_{i,\text{fcfs}} - w)k_i - wd_i) \\
& \quad \Phi_{s^-, \text{fcfs}}(3) 3((r_{s^-, \text{fcfs}} - w)k_{s^-} - wd_{s^-}), \tag{15}
\end{aligned}$$

$$\begin{aligned}
& \tilde{V}(\{\bar{o}_s, \bar{o}_{s^-}\}_{\mathbf{q}_{\text{mkt}, \chi}}, \{(r_{i,\text{mkt}} - w)k_i - wd_i\}_{i=s, s^-}) \equiv \\
& \sum_{i=s, s^-} \bar{\Phi}_{i,\text{mkt}}(2) \left[2((r_{i,\text{mkt}} - w)k_i - wd_i) + (r_{i',\text{mkt}} - w)k_{i'} - wE(d_{i'}) \right] + \\
& \quad \tilde{\Phi}_{i,\text{mkt}}(1_1) ((r_{i,\text{mkt}} - w)k_i - wd_i) + \\
& \quad + \tilde{\Phi}_{i,\text{mkt}}(1_2) ((r_{i,\text{mkt}} - w)k_i - wd_i + (r_{i',\text{fcfs}} - w)k_{i'} - wE(d_{i'})) + \\
& \quad \tilde{\Phi}_{i,\text{mkt}}(2) 2((r_{i,\text{mkt}} - w)k_i - wd_i) \\
& \quad \Phi_{s^-, \text{mkt}}(3) 3((r_{s^-, \text{mkt}} - w)k_{s^-} - wd_{s^-}). \tag{16}
\end{aligned}$$

where the first two terms in either expression correspond to the expected value of serving three orders or different types, while the remaining terms correspond to the expected returns of serving strictly less than three orders.

B Proofs

B.1 Proposition 1

First, we solve for the equilibrium value of service when both farmers queue with both providers. Then, we show the value when the service provider serves a single type of farmers. Then we show that the guess that the selection criteria for the market provider should be to prioritize large scale orders. Finally, we show that the expected value of service is higher for large scale farmers.

Value of service when serving both type of farmers.

Proof. Using the definition of the expected value of service provision (equations 15 and 16) and rearranging terms, as well as the participation constraint for the farmers, equation 4,

the problem of the provider is

$$\begin{aligned} \max_{q_{s,j}, q_{s^-,j}} \sum_{i=s,s^-} \Phi_{i,j}(2) [z_{ij} k_i^\alpha - \frac{U_i}{\Delta_{ij}} - w(k_i + d_i)] + \\ \sum_{i=s,s^-} \Phi_{i,j}(1) [z_{ij} k_{i'}^\alpha - \frac{U_{i'}}{\Delta_{i'j}} - w(k_{i'} + E(d_{i'}))] + \\ \Phi_{s^-,j}(3) 3 [z_{ij} k_{s^-}^\alpha - \frac{U_{s^-}}{\Delta_{s^-j}} - w(k_{s^-} + d_{s^-})], \end{aligned}$$

where $\Phi_{i,j}(2) = (\bar{\Phi}_{i,j}(2)2 + \tilde{\Phi}_{i,j}(2)2 + \tilde{\Phi}_{i,j}(1_1) + \tilde{\Phi}_{i,j}(1_2))$ and $\Phi_{i,j}(1) = (\bar{\Phi}_{i,j}(2) + \tilde{\Phi}_{i,j}(1_2))$.

Let \bar{V}_{ij} be the profit per order of type i for provider j , i.e. $\bar{V}_{ij} \equiv ((r_{i,j} - w)k_i - wd_i)$. The optimality condition with respect to the queue length of large-scale and small-holder farmers are

$$\begin{aligned} \sum_{i=s,s^-} \left(\frac{\partial \Phi_{i,j}(2)}{\partial q_{s,j}} \bar{V}_i + \frac{\partial \Phi_{i,j}(1)}{\partial q_{s,j}} \bar{V}_{i'} \right) + 3 \frac{\partial \Phi_{s^-,j}(3)}{\partial q_{s,j}} \bar{V}_{s^-} + \\ \sum_{i=s,s^-} \left(\Phi_{i,j}(2) \frac{\partial \bar{V}_i}{\partial q_{s,j}} + \Phi_{i,j}(1) \frac{\partial \bar{V}_{i'}}{\partial q_{s,j}} \right) + \Phi_{s^-,j}(3) 3 \frac{\partial \bar{V}_{s^-}}{\partial q_{s,j}} = 0, \end{aligned} \quad (17)$$

$$\begin{aligned} \sum_{i=s,s^-} \left(\frac{\partial \Phi_{i,j}(2)}{\partial q_{s^-,j}} \bar{V}_i + \frac{\partial \Phi_{i,j}(1)}{\partial q_{s^-,j}} \bar{V}_{i'} \right) + 3 \frac{\partial \Phi_{s^-,j}(3)}{\partial q_{s^-,j}} \bar{V}_{s^-} + \\ \sum_{i=s,s^-} \left(\Phi_{i,j}(2) \frac{\partial \bar{V}_i}{\partial q_{s^-,j}} + \Phi_{i,j}(1) \frac{\partial \bar{V}_{i'}}{\partial q_{s^-,j}} \right) + \Phi_{s^-,j}(3) 3 \frac{\partial \bar{V}_{s^-}}{\partial q_{s^-,j}} = 0, \end{aligned} \quad (18)$$

where

$$\frac{\partial \bar{V}_i}{\partial q_{i,j}} = - \left[\frac{\bar{V}_i + w(k_i + d_i) - z_{ij} k_i^\alpha (1 + \frac{\partial z_{ij}}{\partial \Delta_{ij}} \frac{\Delta_{ij}}{z_{ij}})}{\Delta_{ij}} \right] \left(\frac{\partial \Delta_{i,j}}{\partial q_{i,j}} \right).$$

Assumption 3. Let $\bar{\theta}(\Delta_{ij}) = -\ln(\Delta_{ij})$, a strictly decreasing function of the probability of service.

Given Assumption 3 elasticity of productivity to the probability of service equals $-\eta$. We can define

$$\tilde{z} \equiv \bar{z}(1 - \eta),$$

as the ‘‘adjusted’’ productivity, and the envelope condition reads

$$\frac{\partial \bar{V}_i}{\partial q_{i,j}} = -(\bar{V}_i + w(k_i + d_i) - \tilde{z}k_i^\alpha(1 - \eta)) \frac{1}{\Delta_{ij}} \left(\frac{\partial \Delta_{i,j}}{\partial q_{ij}} \right).$$

Let the elasticity of the probability of service with respect to the queue length be $\zeta_{q\Delta}(o) \equiv -\frac{\partial \Delta_{sj}}{\partial q} \frac{q_{sj}}{\Delta_{sj}(o)}$, let the elasticity of the value of service to the queue length be $\zeta_{\bar{V}q} \equiv \frac{\partial \bar{V}}{\partial q_{ij}} \frac{q_{ij}}{\bar{V}}$ and that of the probability of arrival of o orders to the queue length be $\zeta_{q\psi}(o) \equiv \frac{\partial \psi}{\partial q} \frac{q}{\psi(o)}$. The envelope condition indicates that the elasticity of the value of service to the queue length is an inversely proportional function of the elasticity of the probability of service to the queue length, $\zeta_{\Delta ij}$.

$$\zeta_{\bar{V}_i} = \left(1 + \frac{w(k_i + d_i) - \tilde{z}k_i^\alpha}{\bar{V}_i} \right) \zeta_{\Delta ij}.$$

Equations 17 and 18 form a system of linear equations that can be solved for the two unknowns \bar{V}_{s^-} , \bar{V}_s as a function of the queue lengths.³¹

$$\Gamma \begin{bmatrix} \bar{V}_s \\ \bar{V}_{s^-} \end{bmatrix} = a \begin{bmatrix} \tilde{z}k_s^\alpha - w(k_s + d_s) \\ \tilde{z}k_{s^-}^\alpha - w(k_{s^-} + d_{s^-}) \end{bmatrix}$$

where $\Gamma = \begin{bmatrix} \Gamma_1 \Gamma_2 \\ \Gamma_3 \Gamma_4 \end{bmatrix}$, for

$$\Gamma_1 = \frac{\partial \Phi_{s,j}(2)}{\partial q_{s,j}} + \frac{\partial \Phi_{s^-,j}(1)}{\partial q_{s,j}} + \zeta_{\Delta_s q_s} \left(\frac{\Phi_{s,j}(2)}{q_{s,j}} + \frac{\Phi_{s^-,j}(1)}{q_{s,j}} \right)$$

$$\Gamma_2 = \frac{\partial \Phi_{s^-,j}(2)}{\partial q_{s,j}} + \frac{\partial \Phi_{s,j}(1)}{\partial q_{s,j}} + 3 \frac{\partial \Phi_{s^-,j}(3)}{\partial q_{s,j}} + \zeta_{\Delta_s - q_s} \left(\frac{\Phi_{s,j}(1)}{q_{s,j}} + \frac{\Phi_{s^-,j}(2)}{q_{s,j}} + 3 \frac{\Phi_{s^-,j}(3)}{q_{s,j}} \right)$$

$$\Gamma_3 = \frac{\partial \Phi_{s,j}(2)}{\partial q_{s^-,j}} + \frac{\partial \Phi_{s^-,j}(1)}{\partial q_{s^-,j}} + \zeta_{\Delta_s q_{s^-}} \left(\frac{\Phi_{s^-,j}(1)}{q_{s^-,j}} + \frac{\Phi_{s,j}(2)}{q_{s^-,j}} \right)$$

$$\Gamma_4 = \frac{\partial \Phi_{s^-,j}(2)}{\partial q_{s^-,j}} + \frac{\partial \Phi_{s,j}(1)}{\partial q_{s^-,j}} + 3 \frac{\partial \Phi_{s^-,j}(3)}{\partial q_{s^-,j}} + \zeta_{\Delta_s - q_{s^-}} \left(\frac{\Phi_{s,j}(1)}{q_{s^-,j}} + \frac{\Phi_{s^-,j}(2)}{q_{s^-,j}} + 3 \frac{\Phi_{s^-,j}(3)}{q_{s^-,j}} \right)$$

³¹Note that equation 17 reduces to 21 when there are no small-scale orders.

and in the LHS

$$a = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} \zeta_{\Delta_s q_s} \left(\frac{\Phi_{s,j}(2)}{q_{s,j}} + \frac{\Phi_{s^-,j}(1)}{q_{s,j}} \right) & \zeta_{\Delta_s - q_s} \left(\frac{\Phi_{s,j}(1)}{q_{s,j}} + \frac{\Phi_{s^-,j}(2)}{q_{s,j}} + 3 \frac{\Phi_{s^-,j}(3)}{q_{s,j}} \right) \\ \zeta_{\Delta_s q_{s^-}} \left(\frac{\Phi_{s^-,j}(1)}{q_{s^-,j}} + \frac{\Phi_{s,j}(2)}{q_{s^-,j}} \right) & \zeta_{\Delta_s - q_{s^-}} \left(\frac{\Phi_{s,j}(1)}{q_{s^-,j}} + \frac{\Phi_{s^-,j}(2)}{q_{s^-,j}} + 3 \frac{\Phi_{s^-,j}(3)}{q_{s^-,j}} \right) \end{bmatrix}$$

The last vector on the LHS corresponds to the surplus from trade for each farmer type.

Standard matrix algebra implies that expected value to the providers satisfies

$$\bar{V}_s^j = \gamma_{1s}^j (\tilde{z}k_s^\alpha - wk_s - wd_s) + \gamma_{2s}^j (\tilde{z}k_{s^-}^\alpha - wk_{s^-} - wd_{s^-}) \quad (19)$$

$$\bar{V}_{s^-}^j = \gamma_{1s^-}^j (\tilde{z}k_{s^-}^\alpha - wk_{s^-} - d_{s^-}) + \gamma_{2s^-}^j (\tilde{z}k_s^\alpha - wk_s - wd_s) \quad (20)$$

for $\gamma_{1s^-}^j = \frac{\Gamma_1 a_{22} - a_{12} \Gamma_3}{\Gamma_1 \Gamma_4 - \Gamma_2 \Gamma_3}$ and $\gamma_{2s^-}^j = \frac{a_{21} \Gamma_1 - a_{11} \Gamma_3}{\Gamma_1 \Gamma_4 - \Gamma_2 \Gamma_3}$ while $\gamma_{1s}^j = \frac{a_{11} \Gamma_4 - \Gamma_2 a_{21}}{\Gamma_1 \Gamma_4 - \Gamma_2 \Gamma_3}$ and $\gamma_{2s}^j = \frac{a_{12} \Gamma_4 - \Gamma_2 a_{22}}{\Gamma_1 \Gamma_4 - \Gamma_2 \Gamma_3}$. Notice that the denominator of each of the γ parameters shifts depending on the provider as a function of the probability of service. This heterogeneity changes the value for the derivatives in Γ . \square

Value of service when serving only large scale farmers.

If a **provider j attracts only large-scale farmers**, i.e. $q_{s^-j} = 0$, then the expected per period profit of the provider satisfies

$$\bar{V}_s = \gamma(q_{sj}, \zeta_{\Delta q}, \zeta_{\psi q}, \alpha) (\tilde{z}k_s^\alpha - w(k_s + d_s))$$

where the second term corresponds to the surplus associated to the transaction and $\gamma \in (0, 1)$ is a non-linear function of the queue length, the elasticity of the service probability with respect to the length of the queue, ζ , and the share of capital in farming production.

Proof. The problem of the supplier when it only receives large scale orders is

$$\max_{q_{sj}, r_{sj}} \psi \bar{V}_s$$

subject to

$$\begin{aligned} \Delta_{sj} \pi_s(r_{sj}, k_s) &\geq U_s \\ \sum_{i \in \hat{q}_j} k_s(i) + E(d_s(i)) &\leq \bar{k} \end{aligned}$$

where $\psi = 2(1 - e^{-q_{s,\text{mkt}}}(1 + q_{s,\text{mkt}})) + e^{-q_{s,\text{mkt}}} q_{s,\text{mkt}}$ because there are no small-scale orders and either the supplier serves one or two orders of large scale.

Using the definition of profits to the farmers, equation 4, we can replace the cost of capital into the objective function. Replacing the rental price of capital as a function of the expected profits, the provider solves

$$\max_{q_{sj}} \psi \left[\bar{z}k_s^\alpha - \frac{U_s}{\Delta_{sj}} - w(k_s + d_s) \right]$$

Note that the properties of the probabilities ψ and Δ (decreasing and convex in the queue length) imply that the first order conditions to the problem are necessary and sufficient for an optimum. The optimality condition for the queue length is

$$\frac{\partial \psi}{\partial q} \bar{V}_s - \psi \left[\frac{\bar{V}_s + w(k_s + d_s) - \bar{z}k_s^\alpha}{\bar{\Delta}_{sj}(2)} \right] \frac{\partial \bar{\Delta}_{sj}(2)}{\partial q} = 0. \quad (21)$$

Let the elasticity of the probability of service with respect to the queue length be $\zeta_{q\Delta}(o) \equiv -\frac{\partial \Delta_{sj}}{\partial q} \frac{q_{sj}}{\Delta_{sj}(o)}$ and let the elasticity of the probability of arrival of o orders to the queue length be $\zeta_{q\psi}(o) \equiv \frac{\partial \psi}{\partial q} \frac{q}{\psi(o)}$. Finally, let $\gamma(q_{s,j}, \zeta_{\Delta q}, \zeta_{\psi q}, \alpha) \equiv \frac{\zeta_{q\Delta}}{\zeta_{q\psi} + \zeta_{q\Delta}}$,

$$\bar{V}_s = \gamma(q_{sj}, \zeta_{\Delta q}, \zeta_{\psi q}, \alpha) (\bar{z}k_s^\alpha - w(k_s + d_s)) \quad (22)$$

which proves the result. For the value to be positive we require $\gamma > 0$ which is true by construction. The provider takes a fraction of the surplus from the transaction. \square

If a provider j attracts only small-holder farmers, i.e. $q_{s,j} = 0$, then the expected per period profit of the provider satisfies

$$\bar{V}_{s-} = \gamma(q_{s-j}, \zeta_{\Delta q}, \zeta_{\psi q}, \alpha) (\bar{z}k_{s-}^\alpha - w(k_{s-} + d_{s-}))$$

where $\gamma \in (0, 1)$ is a non-linear function of the queue length, the elasticity of the service probability with respect to the length of the queue, ζ , and the share of capital in farming production.

The derivations when the provider serves only small-scale providers follow the same steps as the ones above, so we omit them for brevity.

The market provider wants to prioritize large scale orders: Compute $\frac{\partial \Pi_{\text{mkt}}}{\partial \chi}$, which are strictly positive, given the definition for the unconditional probabilities of service, 12 to 14, and the value of the provider, equations 8 and 16. Then, the optimal selection rule is at the corner, $\chi = 1$.

Expected profits to the farmers The expected profits to the farmers depend on the equilibrium being realized, i.e. whether providers serve both type of farmers or providers specialize in a single type. The reason is that the expected profits to the farmer depend on the cost of service, which can be in turn expressed as a function of the value of service using the definition of the value per period, $\tilde{z}k_i^\alpha - \frac{U_i}{\Delta_{ij}} - w(k_i + d_i) = \bar{V}_i^j$

$$U_i = (\tilde{z}k_i^\alpha - w(k_i + d_i) - \bar{V}_i^j)\Delta_{ij} \quad (23)$$

Replacing the values of expected profits for the providers we obtain

1. If providers serve both type of farms,

$$\begin{aligned} U_s &= \Delta_{sj}((1 - \gamma_{1s}^j)(\tilde{z}k_s^\alpha - wk_s + wd_s) - \gamma_{2s}^j(\tilde{z}k_{s-}^\alpha - wk_{s-} + wd_{s-})) \\ U_{s-} &= \Delta_{s-j}((1 - \gamma_{1s-}^j)(\tilde{z}k_{s-}^\alpha - wk_{s-} + wd_{s-}) - \gamma_{2s-}^j(\tilde{z}k_s^\alpha - wk_s + wd_s)) \end{aligned}$$

2. If a provider serves only large scale farmers,

$$U_s = \tilde{\Delta}_{sj}(1 - \gamma(q_{sj}, \zeta_{\Delta q}, \zeta_{\psi q}, \alpha))(\tilde{z}k_s^\alpha - wk_s + wd_s)$$

3. If a provider serves only small scale farmers,

$$U_{s-} = \tilde{\Delta}_{s-j}(1 - \gamma(q_{s-j}, \zeta_{\Delta q}, \zeta_{\psi q}, \alpha))(\tilde{z}k_{s-}^\alpha - wk_{s-} + wd_{s-})$$

When the providers specialize in service provision, they determine the expected profits to the farmer.

Equilibrium queue lengths. In an equilibrium where farmers reach out to both providers, they should be indifferent across them and the feasibility constraints of the economy should be satisfied.³² We describe the indifference condition for large scale farmers, the ones for small scale farmers are analogous.

$$\frac{\Delta_{\text{smkt}}}{\Delta_{\text{sfcfs}}} = \frac{(1 - \gamma_{1s}^{\text{fcfs}})(\tilde{z}k_s^\alpha - wk_s - wd_s) - \gamma_{2s}^{\text{fcfs}}(\tilde{z}k_{s-}^\alpha - wk_{s-} - wd_{s-})}{(1 - \gamma_{1s}^{\text{mkt}})(\tilde{z}k_s^\alpha - wk_s - wd_s) - \gamma_{2s}^{\text{mkt}}(\tilde{z}k_{s-}^\alpha - wk_{s-} - wd_{s-})}$$

³²If they choose to reach out to a single provider, then the equilibrium queue length is determined by feasibility only, which in turn determines the expected value for farmers.

When small scale farmers queue only with fcfs providers, the indifference condition for the large farmer is

$$\frac{\Delta_{\text{smkt}}}{\Delta_{\text{sfdfs}}} = \frac{(1 - \gamma_{1s}^{\text{fcfs}})(\tilde{z}k_s^\alpha - wk_s - wd_s) - \gamma_{2s}^{\text{fcfs}}(\tilde{z}k_{s-}^\alpha - wk_{s-} - wd_{s-})}{(1 - \gamma_s^{\text{mkt}})(\tilde{z}k_s^\alpha - wk_s - wd_s)}$$

These indifference conditions jointly with the feasibility constraints of the economy, equations 1 and 2, yield the optimal queue lengths by provider and type.

Rental rates The rental rates can be computed from the definition of U once the optimal queues have been solved for.

Value of service for farmers $U_s \geq U_{s-}$ whenever the different in the surplus of service for large-scale providers is large enough. Because in equilibrium the market value of service is the same irrespective of the provider, it is w.l.o.g. to use the values from the fcfs providers.

$$\begin{aligned} U_s - U_{s-} &\geq 0 \\ (\Delta_{sj}(1 - \gamma_{1s}^j) + \Delta_{s-j}\gamma_{2s-}^j)(\tilde{z}k_s^\alpha - wk_s - wd_s) - \\ (\Delta_{sj}\gamma_{2s}^j + \Delta_{s-j}(1 - \gamma_{1s-}^j))(\tilde{z}k_{s-}^\alpha - wk_{s-} - wd_{s-}) &\geq 0 \end{aligned}$$

If the surplus is weakly higher for large scale farmers, $\tilde{z}k_s^\alpha - wk_s - wd_s \geq \tilde{z}k_{s-}^\alpha - wk_{s-} - wd_{s-}$, then it is sufficient that

$$\frac{(\Delta_{sj}(1 - \gamma_{1s}^j) + \Delta_{s-j}\gamma_{2s-}^j)}{(\Delta_{sj}\gamma_{2s}^j + \Delta_{s-j}(1 - \gamma_{1s-}^j))} \geq \frac{(\tilde{z}k_{s-}^\alpha - wk_{s-} - wd_{s-})}{(\tilde{z}k_s^\alpha - wk_s - wd_s)}. \quad (24)$$

The above is a condition on the elasticities of the probability of service to the queue lengths (in γ) relative to the values of the surplus.

Value of service for providers The value of serving large farmers is higher than the value of serving small-farmers for any provider For $\bar{V}_s > \bar{V}_{s-}$ it is sufficient that $(\gamma_{1s} - \gamma_{2s-}) > 0$ and $(\gamma_{2s} - \gamma_{1s-}) > 0$, see equations 19 and . This is the same as

$$a_{11}(\Gamma_4 + \Gamma_3) - a_{21}(\Gamma_2 + \Gamma_1)$$

$$a_{12}(\Gamma_4 + \Gamma_3) - a_{22}(\Gamma_2 + \Gamma_1)$$

If the queue lengths are the same $\Gamma_4 + \Gamma_3 = \Gamma_2 + \Gamma_1$ and also $a_{11} > a_{21}$ and $a_{12} > a_{22}$ because the elasticity of the probability of service to the queue large farmers is higher than for the queue of small farmers. By continuity, if the queue lengths are not too

different the above result holds. Intuitively, the reason is that if there are no systematic differences in travel time across farmers, then the provider’s marginal cost of provision is higher for the smaller farmers and therefore the provider finds them less valuable.

C Numerical Solution and Output

C.1 Value function computation

The value function maps an ordered queue to the expected present value of this queue. Each order i in the queue comprises two dimensions: h_i , the number of hours demanded discretized to 6 bins, and d_i the travel hours to and from the hub that represents a variable cost of service. For a queue length equal to 3, the value function is a mapping from R^6 to R^1 .

$$V(\{(k_1, \nu_1), (k_2, \nu_2), (k_3, \nu_3)\}) : R^6 \rightarrow [0, \infty]$$

The relatively high dimensionality of the problem prompts us to implement the sparse-grid method proposed by Smolyak (1963) (see [Judd et al. \(2014\)](#) for details). The grid points are selected for an approximation level of 2, which results in 85 grid points. We then construct a Smolyak polynomial consisting 85 orthogonal basis functions, which belong to the Chebyshev family. The integration nodes are selected by applying the tensor product rule to the one-dimensional Smolyak grid points at the approximation level of 2. Integration is carried out using Newton-Cotes quadrature.

C.2 Simulations

We simulate the expected wait time and productivity cost under the fcfs arrangement and the market arrangement respectively for three cases: when productivity is uncorrelated, negatively correlated or positively correlated to the number of hours demanded. Productivity, measured in revenue per acre, is simulated and assigned to each order observed in the actual data. We make a large number of draws of productivity sequences, each with length equal to the number of actual orders, from a log normal distribution where the parameters are obtained by fitting the actual productivity information to a log normal distribution by hub. We then choose a sequence for each simulation case that produces a correlation with hours demanded in the data that is the closest to a target correlation for that case, and assign that sequence to the actual orders. In the uncorrelated case, the target is zero; in the negatively correlated case, the target

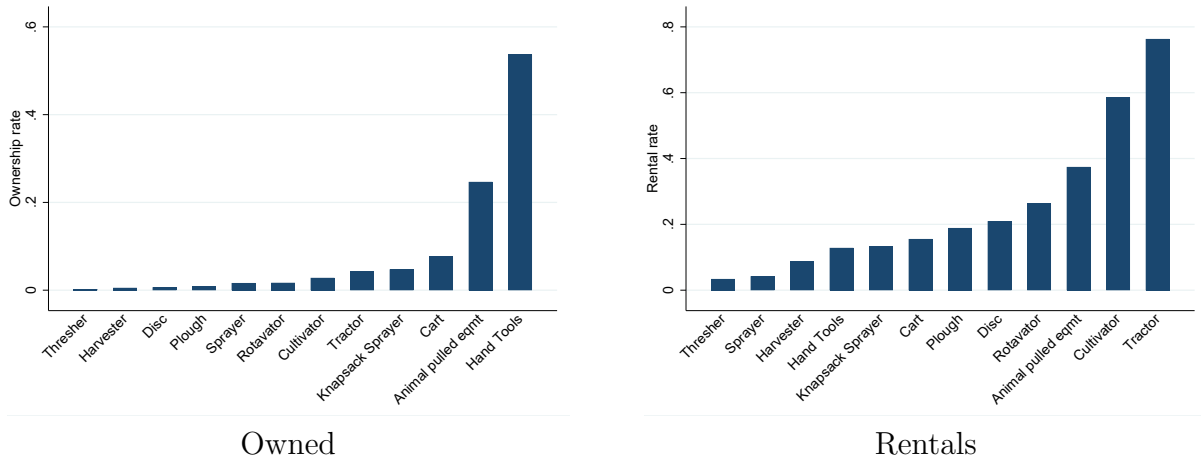
is the actual correlation for that hub; in the positively correlated case, the target is symmetric to the negative correlation.

We use bootstrap sampling of the actual orders for the simulation and assume each bootstrap sample represents an actual queue.

We compute the wait time for the first three orders in each bootstrap sample under the fcfs arrangement and the market arrangement respectively. In any period if one or more out of these three orders are not served, the queue is filled going through the bootstrap sample. The productivity cost is calculated by multiplying the simulated productivity by the wait-time and the percentage productivity loss per day as described in the table 3.

D Additional Tables and Figures

Figure 15: Ownership and rentals by implement.



Notes: The ownership (rental) rate is the share of farmers that report to own a given implement relative to the total population surveyed.

Table 9: Costs of Delays Relative to Optimal Planting Time, Value Added per Acre

	Cost per day, value added per acre				
	Whole Sample	5-day around optimal		10-day around optimal	
		Before	After	Before	After
β_1	-41.97 (26.33)	391.1** (140.2)	-215.7 (146.9)	1,166*** (338.7)	-931.1*** (298.5)
Observations	6,034	1,461	1,882	1,010	1,221
R-squared	0.408	0.625	0.584	0.706	0.659
Mean of Value Added per acre	10228	11425	10694	11921	10998

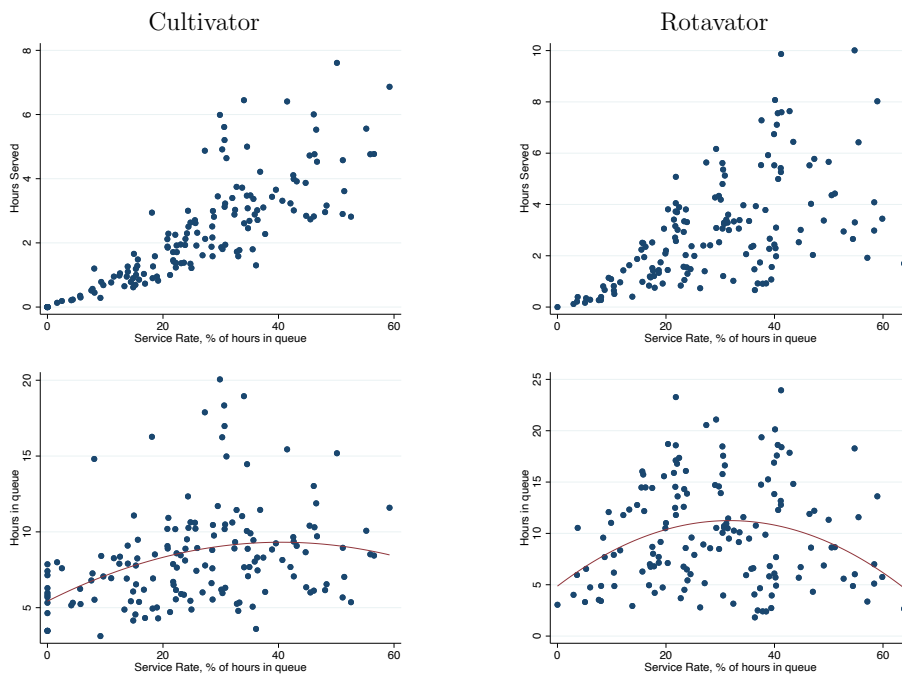
Notes: The optimal date is a village-year measure, and is the sowing date that maximized mean value added per acre in the village in a given year. Standard errors clustered at the village-level in parentheses.
 * $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$, **** $p < 0.001$.

Table 10: Status Quo Equilibrium Outcomes

(a) Average queue length					(b) Rental rates				
	small		large			small		large	
	fcfs	mkt	fcfs	mkt		fcfs	mkt	fcfs	mkt
1	2.0	2.0	1.1	2.6	1	1.00	1.01	0.76	0.76
2	3.0	1.7	2.1	2.7	2	0.84	0.89	0.73	0.74
3	3.5	1.5	1.1	3.0	3	1.04	1.09	0.82	0.82
4	3.3	1.8	1.2	2.4	4	0.92	0.97	0.73	0.74
5	4.3	2.0	1.1	2.5	5	0.95	0.98	0.74	0.74
6	2.0	2.2	1.1	2.2	6	1.10	1.11	0.81	0.81
7	2.0	2.8	1.0	1.7	7	1.22	1.23	0.85	0.86
8	2.0	1.9	1.1	2.6	8	1.04	1.06	0.75	0.76
9	2.5	1.3	1.1	3.6	9	0.92	0.99	0.74	0.74
10	4.0	1.6	1.2	3.9	10	0.90	0.93	0.68	0.69
11	4.0	1.9	1.0	2.3	11	0.96	1.02	0.75	0.78

Notes: Panel (a) reports equilibrium average queue lengths by hub for farmers of different scale and different providers. Panel (b) reports the equilibrium rental rates per hour for each provider and service type within each market. Rental rates are normalized to the rental rate charged by the fcfs provider to small-scale farmers in hub 1.

Figure 16: Service rates.



Notes: Service rates and queuing behavior in CHCs, Kharif 2018. Each data point corresponds to the average queue or hours served across hubs per day in the season. The bottom panels include a polynomial fit of the series. Source: Own computations and CHC admin data.

Figure 17: Empirical distribution of location and demand hours

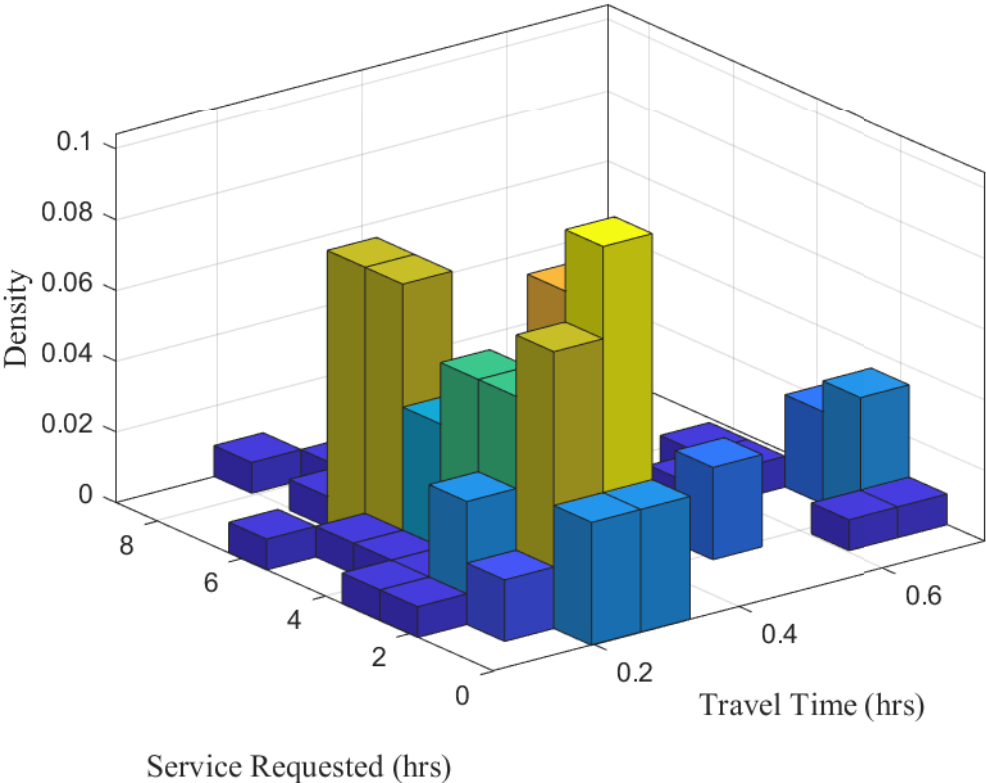
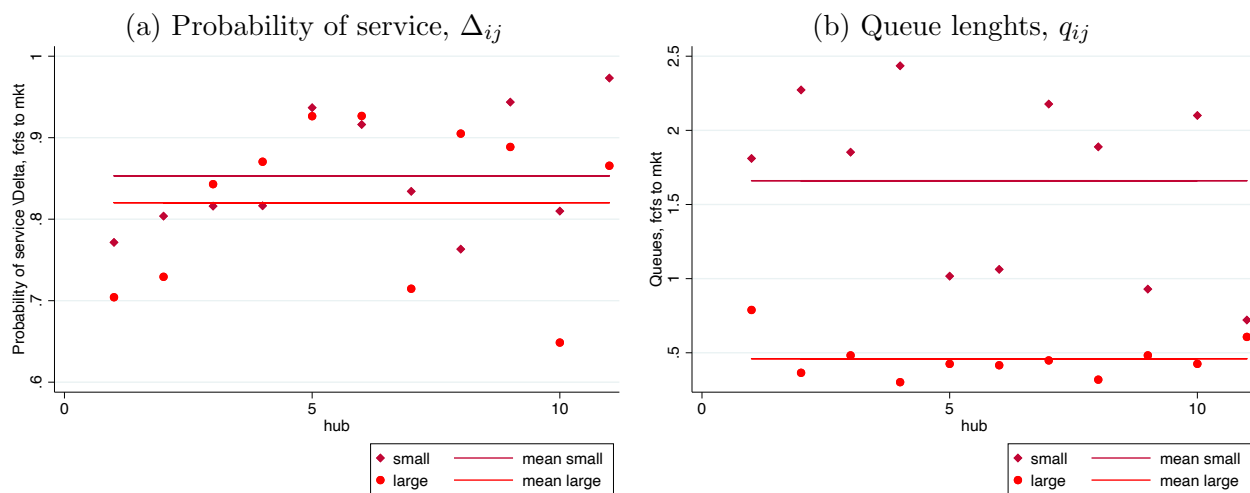


Figure 18: Service finding rate decomposition

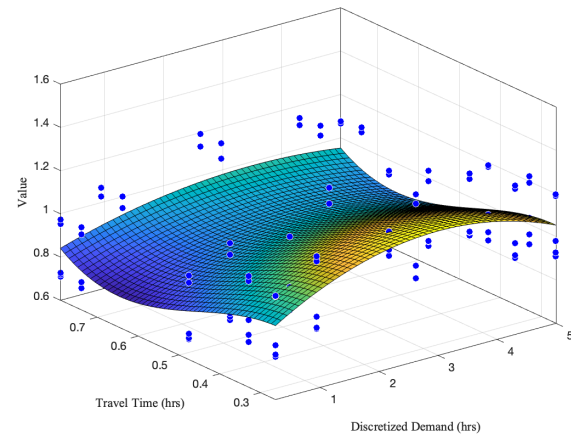
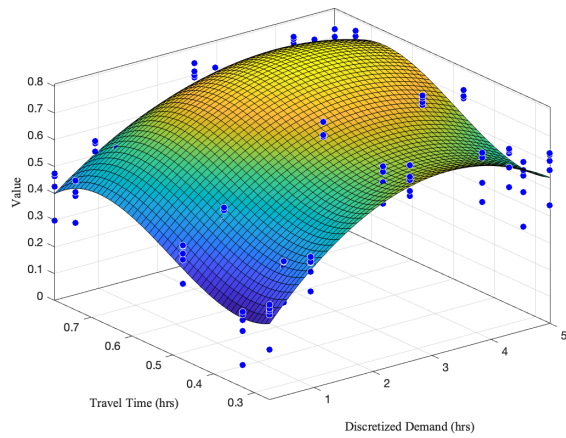
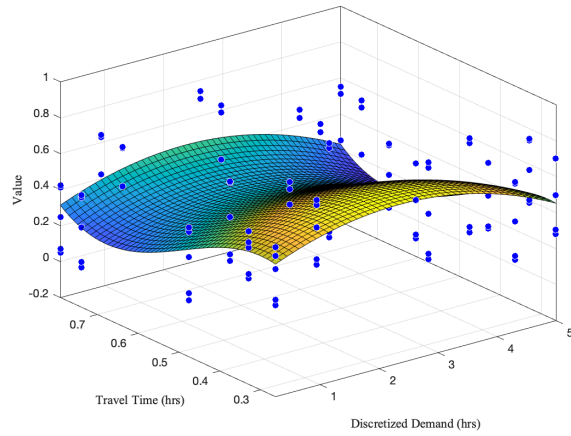
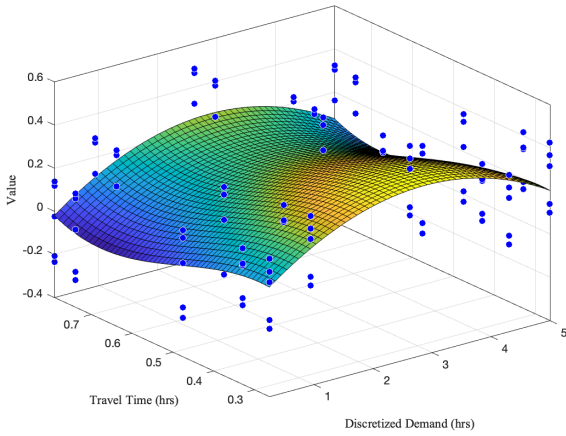


Notes: Conditional probability of service and queue lengths by hub for the calibrated economy.

Figure 19: Value Functions

Benchmark

TSP



Notes: Value of service provision, economy with full heterogeneity. Blue dots corresponds to the nodes for the tensor product interpolation as described in Appendix C. The left panel corresponds to the benchmark economy, while the right panel corresponds to the economy where service providers optimize service in space, i.e. solve a traveling salesman problem, “TSP”. The top panel correspond to the value of the FCFS providers while the bottom panel corresponds to the market provider.